Chapter - 1

NCERT CORNER

Intext Questions

1. Does every number have an even number of factors?

No.

Most numbers have factors in pairs.

For example:

12

Factors: 1,2,3,4,6,12

Factor pairs:

$$1 \times 12$$
$$2 \times 6$$
$$3 \times 4$$

Total factors = 6 (even)

But **perfect squares** have one factor repeated in the middle.

Example:

$$36 = 6 \times 6$$

So 6 pairs with itself.

Hence, perfect squares have an **odd number of factors**.

Therefore,

$$\boxed{\text{No, every number does not have an even number of factors.}}$$

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2. Check whether 36 has an odd number of factors.

Factors of 36:

$$1,\ 2,\ 3,\ 4,\ 6,\ 9,\ 12,\ 18,\ 36$$

Number of factors:

$$9$$

Since 9 is odd,

$$\boxed{\text{36 has an odd number of factors.}}$$

Factor pairs:

$$1 \times 36$$
$$2 \times 18$$
$$3 \times 12$$
$$4 \times 9$$
$$6 \times 6$$

All factors except 6 have different partners. The factor 6 pairs with itself.

Hence the statement is **true**.

Intext Questions

Question 1.
Find the squares of the first 30 natural numbers and fill in the table below.

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 =$	$22^2 =$
$3^2 = 9$	$13^2 =$	
$4^2 = 16$	$14^2 =$	
$5^2 = 25$	$15^2 =$	
$6^2 =$	$16^2 =$	
$7^2 =$	$17^2 =$	
$8^2 =$	$18^2 =$	
$9^2 =$	$19^2 =$	
$10^2 =$	$20^2 =$	

Solution:

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$

Question 2.
What patterns do you notice? Share your observations and make conjectures.
Solution:
In the above sequence, we notice that the sum of consecutive odd numbers is a perfect square.
We see that the next perfect square is the sum of the next consecutive odd number.
Observation:
$1^2 = 1$
$2^2 = 1 + 3 = 4$
$3^2 = 1 + 3 + 5 = 9$
$4^2 = 1 + 3 + 5 + 7 = 16$
$5^2 = 16 + 9 = 25$
$6^2 = 25 + 11 = 36$
$7^2 = 36 + 13 = 49$
.
.
.
.
$28^2 = 729 + 55 = 784$
$29^2 = 784 + 57 = 841$
$30^2 = 841 + 59 = 900$

IN TEXT

1. Which of the following numbers has the digit 6 in the units place?
(i) 38^2
(ii) 34^2
(iii) 46^2
(iv) 56^2
(v) 74^2
(vi) 82^2
Solwer:
We know if a number has 4 or 6 in the units place, then its square ends in 6.
So, squares of (ii) 34, (iii) 46, (iv) 56, and (v) 74 have the digit 6 in the units place.

Question 2.
If a number contains 3 zeros at the end, how many zeros will its square have at the end?
Solution:
If a number contains 3 zeros at the end, then its square will have 6 zeros at the end.

Question 3.
What do you notice about the number of zeros at the end of a number and the number of zeros at the end of its square? Will this always happen? Can we say that squares can only have an even number of zeros at the end? (Page 5)
Solution:
We noticed that the number of zeros at the end of its square has doubled.
Yes, it will always happen.
And also, we can say that squares can only have an even number of zeros at the end.

Question 4.
What can you say about the parity of a number and its square? (Page 5)
Solution:
We say that when a number is multiplied by itself, it is called its square.

IN TEXT

1. Find more such patterns by observing the numbers and their squares from the table you filled earlier.
Solution:
If a number has a 3 or 9 in the units place, its square will always end in 6.
If a number has a 2 or 8 in the units place, its square will always end in 4.
If a number has a 5 in the units place, its square will always end in 5.

Question 2: Square Numbers Between 1 and 1000

Task: Count how many perfect square numbers fall in each block of 100. Also find the largest perfect square less than 1000.

Key Concept

A perfect square is the product of an integer multiplied by itself ($n \times n = n^2$).

Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024...

Range	Perfect Squares in Range	Count	Note
1 – 100	1 (1^2), 4 (2^2), 9 (3^2), 16 (4^2), 25 (5^2), 36 (6^2), 49 (7^2), 64 (8^2), 81 (9^2), 100 (10^2)	10	$1^2=1$ to $10^2=100$
101 – 200	121 (11^2), 144 (12^2), 169 (13^2), 196 (14^2)	4	$11^2=121$ to $14^2=196$
201 – 300	225 (15^2), 256 (16^2), 289 (17^2)	3	$15^2=225$ to $17^2=289$
301 – 400	324 (18^2), 361 (19^2), 400 (20^2)	3	$18^2=324$ to $20^2=400$
401 – 500	441 (21^2), 484 (22^2)	2	$21^2=441$ to $22^2=484$
501 – 600	529 (23^2), 576 (24^2)	2	$23^2=529$ to $24^2=576$
601 – 700	625 (25^2), 676 (26^2)	2	$25^2=625$ to $26^2=676$
701 – 800	729 (27^2), 784 (28^2)	2	$27^2=729$ to $28^2=784$
801 – 900	841 (29^2), 900 (30^2)	2	$29^2=841$ to $30^2=900$
901 – 1000	961 (31^2)	1	$31^2=961$; $32^2=1024>1000$

Summary Answers

Between 1 and 100: 10 perfect squares ($1^2 = 1$ through $10^2 = 100$)

There are 10 perfect square numbers between 1 and 100.

Between 101 and 200: 4 perfect squares ($11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$)

There are 4 perfect square numbers between 101 and 200.

Largest Perfect Square Less Than 1000

We need the largest n such that $n^2 < 1000$:

$31^2 = 31 \times 31 = 961 \quad \rightarrow \quad 961 < 1000$ ✓

$32^2 = 32 \times 32 = 1024 \quad \rightarrow \quad 1024 > 1000$ ✗

Largest perfect square less than 1000 is 961 (= 31^2).

Pattern: Each block of 100 has fewer perfect squares as numbers grow larger, because squares spread apart as n increases.

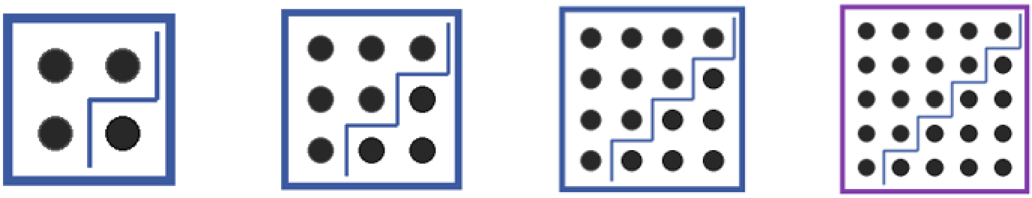
Question 3: Relation Between Triangular Numbers and Square Numbers

What are Triangular Numbers?

Triangular numbers: $T_1=1$, $T_2=3$, $T_3=6$, $T_4=10$, $T_5=15$ ... Formula: $T_n = n(n+1) \div 2$

The Pattern from the Diagram

Every square number equals the SUM of two consecutive triangular numbers.



Term 1
$1 + 3 = 4 = 2^2$
$T_1 + T_2$

Term 2
$3 + 6 = 9 = 3^2$
$T_2 + T_3$

Term 3
$6 + 10 = 16 = 4^2$
$T_3 + T_4$

Term 4 (NEXT TERM)
$10 + 15 = 25 = 5^2$
$T_4 + T_5$

Step-by-Step Explanation of Each Term

Term	T_n (smaller)	T_{n+1} (larger)	Sum = Square Number	Grid
1	$T_1 = 1$	$T_2 = 3$	$1 + 3 = 4 = 2^2$	2 × 2 dot grid
2	$T_2 = 3$	$T_3 = 6$	$3 + 6 = 9 = 3^2$	3 × 3 dot grid
3	$T_3 = 6$	$T_4 = 10$	$6 + 10 = 16 = 4^2$	4 × 4 dot grid
4 (Next)	$T_4 = 10$	$T_5 = 15$	$10 + 15 = 25 = 5^2$	5 × 5 dot grid

Finding the Next Term

The 4th term uses T_4 and T_5:

$$T_4 = 4 \times (4 + 1) \div 2 = 4 \times 5 \div 2 = 10$$

$$T_5 = 5 \times (5 + 1) \div 2 = 5 \times 6 \div 2 = 15$$

$T_4 + T_5 = 10 + 15 = 25 = 5^2$

Next term: $10 + 15 = 25 = 5^2$ (shown as a 5×5 dot grid)

The Relation (General Rule)

General Formula   Example Verification

$n^2 = T_{n-1} + T_n$ (every square = sum of two consecutive triangular numbers) $5^2 = T_4 + T_5 = 10 + 15 = 25$ ✓

The staircase line in each dot diagram shows exactly how the two triangular numbers fit together to form the square.

IN TEXT

(i) **1156**

2	1156
2	578
17	289
17	17
	1

$\therefore 1156 = 2 \times 2 \times 17 \times 17$

$= 2^2 \times 17^2$

$= (2 \times 17)^2$

$= 34^2$

∴ 1156 is a perfect square.

(ii) **2800**

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

$\therefore 2800 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$

$= 2^4 \times 5^2 \times 7$

∴ 2800 is not a perfect square. (Since 7 is unpaired)

Figure It Out

Question 1.
Which of the following numbers are not perfect squares?
(i) 2032
(ii) 2048
(iii) 1027
(iv) 1089
Solution:
(i) 2032 is not a perfect square, as a number ending with 2 can not be a perfect

square.
(ii) 2048 is not a perfect square, as a number ending with 8 can not be a perfect square.

(iii) 1027 is not a perfect square, as a number ending with 7 can not be a perfect square.
(iv) 1089 ends in 9 at the unit's place. Hence, it is a perfect square.

Question 2.
Which one among 64^2, 108^2, 292^2, 36^2 has the last digit 4?
Solution:
(i) Unit's digit of 64 is 4
$\therefore 4^2 = 4 \times 4 = 16$ (last digit = 6)
(ii) Unit's digit of 108 is 8
$\therefore 8^2 = 8 \times 8 = 64$ (last digit = 4)
(iii) Unit's digit of 292 is 2
$\therefore 2^2 = 2 \times 2 = 4$
(iv) Unit's digit of 36 = 6
$\therefore 6^2 = 6 \times 6 = 36$ (last digit = 6)
Hence, the numbers whose squares end in 4 are 108^2 and 292^2.

Question 3.
Given $125^2 = 15625$, what is the value of 126^2?
(i) 15625 + 126
(ii) $15625 + 26^2$
(iii) 15625 + 253
(iv) 15625 + 251
(v) $15625 + 51^2$
Solution:
Here, $126^2 = (125 + 1)^2$
$= (125)^2 + 2 \times 125 \times 1 + (1)^2$ [Using identity $(a + b)^2 = a^2 + 2ab + b^2$]
= 15625 + 250 + 1
= 15625 + 251
So, the value of 126^2 is (iv) option, i.e., 15625 + 251.

Question 4.
Find the length of the side of a square whose area is 441 m^2.
Solution:

3	441
3	147
7	49
7	7
	1

Area of square = side × side = 441
⇒ $side^2 = 441$
⇒ side = √441
441 = 3 × 3 × 7 × 7
√444 = 3 × 7 = 21
∴ Side of square = 21 m.

Question 5.
Find the smallest square number that is divisible by each of the following numbers: 4, 9, and 10.
Solution:
To find the required smallest square number, we will find the least number divisible by each of 4, 9, and 10, i.e., LCM of 4, 9, and 10.

2	4, 9, 10
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

LCM = 2 × 2 × 3 × 3 × 5 = 180
Prime factorisation of 180 = 2 × 2 × 3 × 3 × 5
5 is not in pairs, so 180 is not a square number.
In order to get a perfect square, we will multiply 180 by 5.
So, the required smallest square number is 900.

Question 6.
Find the smallest number by which 9408 must be multiplied so that the product is a perfect square. Find the square root of the product.
Solution:

2	9408
2	4704
2	2352
2	1176
3	588
3	294
3	147
7	49
7	7
	1

$9408 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7$

All prime factors of 9408 are arranged in pairs except 3.

So, we multiply 9408 by 3 to make it a perfect square.

Perfect square = $9408 \times 3 = 28224$

Now,

and 28224is a perfect square, let's find its square root using prime factorization:

Prime factorization of 28224

2	28224	$28224 \div 2 = 14112$
2	14112	$14112 \div 2 = 7056$
2	7056	$7056 \div 2 = 3528$
2	3528	$3528 \div 2 = 1764$
2	1764	$1764 \div 2 = 882$
2	882	$882 \div 2 = 441$
3	441	$441 \div 3 = 147$
3	147	$147 \div 3 = 49$
7	49	$49 \div 7 = 7$
7	7	$7 \div 7 = 1$
	1	

$28224 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$28224 = 2^6 \times 3^2 \times 7^2$

So,

$$\sqrt{28224} = \sqrt{2^6 \times 3^2 \times 7^2} = 2^3 \times 3 \times 7 = 8 \times 3 \times 7 = 168$$

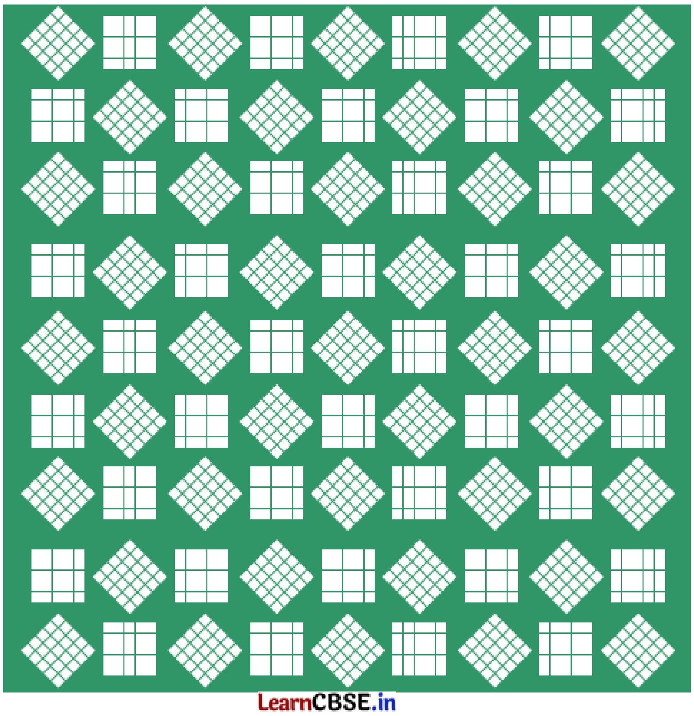
Therefore,

$$\boxed{\sqrt{28224} = 168}$$

Question 7.
How many numbers lie between the squares of the following numbers?
(i) 16 and 17
(ii) 99 and 100
Solution:
(i) Numbers lying between 16^2 and $17^2 = 2 \times 16 = 32$
(ii) Numbers lying between 99^2 and $100^2 = 2 \times 99 = 198$

Question 8.
In the following pattern, fill in the missing numbers:
$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + 12^2 = 13^2$
$4^2 + 5^2 + 20^2 = (\_\_)2$
$9^2 + 10^2 + (\_\_)2 = (\_\_)2$
Solution:
$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + 12^2 = 13^2$
$4^2 + 5^2 + 20^2 = (21)^2$
$9^2 + 10^2 + (90)^2 = (91)^2$

Question 9.
How many tiny squares are there in the following picture? Write the prime factorisation of the number of tiny squares.



Solution:
Big squares in a row = 9
Big squares in a column = 9
Tiny squares in a big square = 25
∴ Total tiny squares = 9 × 9 × 25 = 2025
Now prime factorisation of 2025 = 3 × 3 × 3 × 3 × 5 × 5 = 452

IN TEXT

1. How many cubes of side 1 cm make a cube of side 2 cm?

Number of small cubes

$$= \left(\frac{2}{1}\right)^3 = 2^3 = 8$$

Answer: 8 cubes

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2. How many cubes of side 1 cm make a cube of side 3 cm?

$$= \left(\frac{3}{1}\right)^3 = 3^3 = 27$$

Answer: 27 cubes

3(i). Complete the table

Number	Cube
1^3	1
2^3	8
3^3	27
4^3	64
5^3	125
6^3	216
7^3	343
8^3	512
9^3	729
10^3	1000
11^3	1331
12^3	1728
13^3	2197
14^3	2744
15^3	3375
16^3	4096
17^3	4913
18^3	5832
19^3	6859
20^3	8000

3(ii). What patterns do you notice?

1. Cubes ending in **1, 4, 5, 6, 9, 0** have the same last digit.
2. Cubes can end only in **0, 1, 4, 5, 6, 8, 9, 2, 3, 7**.
3. Consecutive cubes increase rapidly.

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4. What are the possible last digits of cubes?

Number ends in	Cube ends in
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Therefore, cubes can end with any digit from 0 to 9.

5. Number of cubes with 1, 2 and 3 digits

1-digit cubes:

$$1, 8$$

Number = **2**

2-digit cubes:

$$27, 64$$

Number = **2**

3-digit cubes:

$$125,216,343,512,729$$

Number = **5**

Observation: As numbers increase, cubes gain more digits rapidly.

6. Can a cube end with exactly two zeros (00)?

No.

A cube ending in 00 would require factors of 10^2.

But powers in a perfect cube occur in multiples of 3.

Hence a cube cannot have exactly two trailing zeros.

Answer: No.

7. Express 4104 and 13832 as sum of two positive cubes

4104

$$2^3 + 16^3 = 8 + 4096 = 4104$$

Also,

$$9^3 + 15^3 = 729 + 3375 = 4104$$

$$\boxed{4104 = 2^3 + 16^3 = 9^3 + 15^3}$$

13832

$$2^3 + 24^3 = 8 + 13824 = 13832$$

Also,

$$18^3 + 20^3 = 5832 + 8000 = 13832$$

$$\boxed{13832 = 2^3 + 24^3 = 18^3 + 20^3}$$

IN TEXT

1. Find the cube roots of the following numbers

(i) $\sqrt[3]{64}$

We know that:

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Grouping the factors in threes:

$$64 = (2 \times 2 \times 2)(2 \times 2 \times 2)$$
$$64 = 2^3 \times 2^3$$
$$64 = (2 \times 2)^3$$
$$64 = 4^3$$

Therefore,

$$\sqrt[3]{64} = \sqrt[3]{4^3}$$
$$\boxed{\sqrt[3]{64} = 4}$$

(ii) $\sqrt[3]{512}$

Prime factorisation:

$$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

Grouping the factors in threes:

$$512 = (2 \times 2 \times 2)(2 \times 2 \times 2)(2 \times 2 \times 2)$$
$$512 = 2^3 \times 2^3 \times 2^3$$
$$512 = (2 \times 2 \times 2)^3$$
$$512 = 8^3$$

Therefore,

$$\sqrt[3]{512} = \sqrt[3]{8^3}$$
$$\boxed{\sqrt[3]{512} = 8}$$

(iii) ∛729

Prime factorisation:

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Grouping the factors in threes:

$$729 = (3 \times 3 \times 3)(3 \times 3 \times 3)$$
$$729 = 3^3 \times 3^3$$
$$729 = (3 \times 3)^3$$
$$729 = 9^3$$

Therefore,

$$\sqrt[3]{729} = \sqrt[3]{9^3}$$
$$\boxed{\sqrt[3]{729} = 9}$$

2. Compute Successive Differences

Perfect cubes:

$$1, 8, 27, 64, 125, 216$$

First Differences

$$8 - 1 = 7$$
$$27 - 8 = 19$$
$$64 - 27 = 37$$
$$125 - 64 = 61$$
$$216 - 125 = 91$$

First differences:

$$7, 19, 37, 61, 91$$

Second Differences

$$19 - 7 = 12$$
$$37 - 19 = 18$$
$$61 - 37 = 24$$
$$91 - 61 = 30$$

Second differences:

$$12, 18, 24, 30$$

Third Differences

$$18 - 12 = 6$$
$$24 - 18 = 6$$
$$30 - 24 = 6$$

Third differences:

$$6, 6, 6$$

Figure It Out

Question 1.
Find the cube roots of 27000 and 10648.
Solution:
Here,

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$\therefore 27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$
$\therefore = 2 \times 3 \times 5 = 30$

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$\therefore 10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$
$\therefore = 2 \times 11 = 22$

Question 2.
What number will you multiply by 1323 to make it a cube number?
Solution:
Here,

3	1323
3	441
3	147
7	49
7	7
	1

$1323 = 3 \times 3 \times 3 \times 7 \times 7$
To complete the triplet, one more 7 is required.
So, 1323 will be multiplied by 7 to make it a cube number.
So, the cube number = $1323 \times 7 = 9261$
Hence, required number = 7

Question 3.
State true or false. Explain your reasoning.
(i) The cube of any odd number is even.
(ii) There is no perfect cube that ends with 8.
(iii) The cube of a 2-digit number may be a 3-digit number.

(iv) The cube of a 2-digit number may have seven or more digits.
(v) Cube numbers have an odd number of factors.
Solution:
(i) The cube of any odd number is even. (False)
Reason: The cube of an odd number is always odd, as
$3^3 = 27$
$5^3 = 125$
$7^3 = 343$

(ii) There is no perfect cube that ends with 8. (False)
Reason: The cubes of all the numbers ending with 2 at the unit place end with 8.
$2^3 = 8$
$12^3 = 1728$
$22^3 = 10648$

(iii) The cube of a 2-digit number may be a 3-digit number. (False)
Reason: Cube of a 2-digit number may have a minimum of 4 digits to a maximum of 6 digits.
10 is the smallest 2-digit number, and $10^3 = 1000$, which has 4 digits.

(iv) The cube of a 2-digit number may have seven or more digits. (False)
Reason: Cube of a 2-digit number may have at most 6 digits.
99 is the largest 2-digit number, and $99^3 = 970299$, which is a 6-digit number.

(v) Cube numbers have an odd number of factors. (False)
Reason: Cube numbers may have an odd as well as an even number of factors.
As $27 = 3 \times 3 \times 3$ (odd no. of factors)
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (even no. of factors)

Question 4.
You are told that 1331 is a perfect cube. Can you guess without factorisation what its cube root is? Similarly, guess the cube roots of 4913, 12167, and 32768.
Solution:
To find the cube root of 1331
1331
We divide the given number 1331 into two groups, starting from the right side, taking three digits in group 1.
331 → group 1
1 → group 2
331 → unit digit is 1
Hence, the cube roots of one’s digit is 1 …..(1)

Group 2, i.e., 1 only, which is 1^3.
So, the cube roots of one's digit is 1. ……(2)
∵ = 11

4913
Group 1 – 913
Group 2 – 4
Unit digit of 913 is 3.
We know that 3 comes at the unit's place when its cube root ends in 7, as 7 × 7 × 7 = 343
So the unit digit of the cube root of 4913 = 7 …..(1)
Group 2 – 4
4 lies between 1 (i.e., 13) and 23 (i.e., 8)
$1^3 < 4 < 2^3$
Taking the lower limit, the tens digit of the cube root of 4913 is 1. …..(2)
= 17 (from (1) & (2))

12167
Group 1 – 167
Unit digit = 7
So unit digit of cube root of 12167 = 3 as 3 × 3 × 3 = 27
Group 2 – 12
8 < 12 < 27
$2^3 < 12 < 3^3$
Taking the lower limit, the ten's digit of cube root = 2
So = 23

32768
Group 1 – 768
Unit digit = 8
So unit digit of cube root of 32768 = 2 …..(1)
as 2 × 2 × 2 = 8
⇒ = 2
From Group 2 – 32
27 < 32 < 64
$3^3 < 32 < 4^3$
Taking lower limit, ten's digit of the cube root of 32768 is 3. …..(2)
∴ = 32 (from (1) & (2))

Question 5.
Which of the following is the greatest? Explain your reasoning.
(i) $67^3 - 66^3$
(ii) $43^3 - 42^3$
(iii) $67^2 - 66^2$
(iv) $43^2 - 42^2$
Solution:
(i) $67^3 - 66^3 = 1 + 67 \times 66 \times 3$
(ii) $43^3 - 42^3 = 1 + 43 \times 42 \times 3$
(iii) $67^2 - 66^2 = 67 + 66 = 133$
(iv) $43^2 - 42^2 = 43 + 42 = 85$
From above we can see that $67^3 - 66^3$ is the greatest as
$(n + 1)^3 - n^3 = 1 + (n + 1) \times 3n$
$(n + 1)^2 - n^2 = n + n + 1 = 2n + 1$

Practice 1.1

1. Write the pair of partner factors of 12.

Solution By Steps

Step 1: Find all factors of 12

Factors of 12 are:

1, 2, 3, 4, 6, 12

Step 2: Form factor pairs whose product is 12

- $1 \times 12 = 12$
- $2 \times 6 = 12$
- $3 \times 4 = 12$

Final Answer

Partner factors of 12 are:

(1, 12), (2, 6), (3, 4)

2(i). Find the perfect square numbers between 10 and 20.

Solution By Steps

Step 1: List square numbers near 10 and 20

$$n^2$$

- $3^2 = 9$
- $4^2 = 16$
- $5^2 = 25$

Step 2: Check which lies between 10 and 20

Only 16 lies between 10 and 20.

Final Answer

16

2(ii). Find the perfect square numbers between 40 and 50.

Solution By Steps

Step 1: Find nearby square numbers

- $6^2 = 36$
- $7^2 = 49$
- $8^2 = 64$

Step 2: Check which lies between 40 and 50

49 lies between 40 and 50.

Final Answer

49

3(i). Check whether 144 is a perfect square or not.

Solution By Steps

Step 1: Find the prime factors of 144 by division.

Prime factorisation of 144 by division method

2	144
2	72
2	36
2	18
3	9
3	3
	1

Therefore,

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

Step 2: Make pairs of equal factors.

$$144 = (2 \times 2)(2 \times 2)(3 \times 3)$$

All prime factors are paired.

Step 3: Find the square root.

Taking one factor from each pair:

$$\sqrt{144} = 2 \times 2 \times 3$$
$$= 12$$

Step 4: Conclusion

Since all prime factors occur in pairs and the square root is a whole number,

$$144 = 12^2$$

Hence, 144 is a perfect square.

Final Answer

$$\boxed{144 \textit{ is a perfect square.}}$$

$$\boxed{\sqrt{144} = 12}$$

3(ii). Check whether 288 is a perfect square or not.

Solution By Steps

Step 1: Find the prime factorisation of 288.

Prime factorisation of 288

2	288	288 ÷ 2 = 144
2	144	144 ÷ 2 = 72
2	72	72 ÷ 2 = 36
2	36	36 ÷ 2 = 18
2	18	18 ÷ 2 = 9
3	9	9 ÷ 3 = 3
3	3	3 ÷ 3 = 1
	1	

Therefore,

$$\begin{aligned} 288 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^5 \times 3^2 \end{aligned}$$

Step 2: Make pairs of equal factors.

$$288 = (2 \times 2)(2 \times 2)(3 \times 3) \times 2$$

One factor 2 remains unpaired.

Step 3: Conclusion

Since all prime factors are not paired, 288 is not a perfect square.

Final Answer

$$\boxed{\text{288 is not a perfect square.}}$$

3(iii). Check whether 169 is a perfect square or not.

Solution By Steps

Step 1: Find the prime factorisation of 169.

Prime factorisation of 169

Prime factor	Quotient	
13	169	$169 \div 13 = 13$
13	13	$13 \div 13 = 1$
	1	

Therefore,

$$169 = 13 \times 13$$

Step 2: Make pairs of equal factors.

$$169 = (13 \times 13)$$

All factors are paired.

Step 3: Find the square root.

$$\sqrt{169} = 13$$

Step 4: Conclusion

Since all prime factors occur in pairs, 169 is a perfect square.

Final Answer

$$\boxed{\text{169 is a perfect square.}}$$

$$\boxed{\sqrt{169} = 13}$$

---

4. Which of the following numbers are not perfect squares?

(i) 33333

Solution By Steps

Step 1: Observe the unit digit.

33333 ends in 3.

Step 2: Use the property of perfect squares.

A perfect square can never end in 2, 3, 7, or 8.

Final Answer

$$\boxed{\text{33333 is not a perfect square.}}$$

(ii) 26623

Solution By Steps

Step 1: Observe the unit digit.

26623 ends in 3.

Step 2: Apply the property.

A perfect square cannot have 3 in the units place.

Final Answer

$$\boxed{\text{26623 is not a perfect square.}}$$

(iii) 1026000

Solution By Steps

Step 1: Count the zeros at the end.

1026000 has 3 trailing zeros.

Step 2: Apply the property.

The number of zeros at the end of a perfect square is always even.

Since 3 is odd, the number cannot be a perfect square.

Final Answer

$$\boxed{1026000 \text{ is not a perfect square.}}$$

(iv) 152399025

Solution By Steps

Step 1: Observe the last two digits.

The number ends in 25.

A perfect square may end in 25.

Step 2: Verify by finding the square root.

$$3905 \times 3905$$
$$= 152399025$$

Therefore,

$$152399025 = (3905)^2$$

Step 3: Conclusion

Since it is the square of a whole number, it is a perfect square.

Final Answer

152399025 is a perfect square.

Therefore, it is not included among the numbers that are not perfect squares.

5. Find the unit digit of the square of the following numbers.

(i) 29

Unit digit = 9

$9^2 = 81$

Unit digit = 1

Answer: 1

(ii) 132

Unit digit = 2

$2^2 = 4$

Answer: 4

(iii) 64293

Unit digit = 3

$3^2 = 9$

Answer: 9

(iv) 236601

Unit digit = 1

$1^2 = 1$

Answer: 1

(v) 929209

Unit digit = 9

$9^2 = 81$

Unit digit = 1

Answer: 1

(vi) 20121208

Unit digit = 8

$8^2 = 64$

Unit digit = 4

Answer: 4

6. The square of which of the following would be odd numbers?

Property Used

A number and its square have the same parity:

$$\text{Odd}^2 = \text{Odd}, \text{Even}^2 = \text{Even}$$

Therefore:

- If a number is odd, its square is odd.
- If a number is even, its square is even.

(i) 141

Solution By Steps

Step 1: Check whether the number is odd or even.

141 ends in 1.

Therefore, 141 is an odd number.

Step 2: Apply the property.

The square of an odd number is always odd.

Final Answer

$$\boxed{141^2 \text{ is an odd number.}}$$

(ii) 292

Solution By Steps

Step 1: Check whether the number is odd or even.

292 ends in 2.

Therefore, 292 is an even number.

Step 2: Apply the property.

The square of an even number is always even.

Final Answer

$$\boxed{292^2 \text{ is not an odd number. It is even.}}$$

(iii) 1419

Solution By Steps

Step 1: Check whether the number is odd or even.

1419 ends in 9.

Therefore, 1419 is an odd number.

Step 2: Apply the property.

The square of an odd number is always odd.

Final Answer

$$\boxed{1419^2 \text{ is an odd number.}}$$

(iv) 67042

Solution By Steps

Step 1: Check whether the number is odd or even.

67042 ends in 2.

Therefore, 67042 is an even number.

Step 2: Apply the property.

The square of an even number is always even.

Final Answer

$$\boxed{67042^2 \text{ is not an odd number. It is even.}}$$

Answer

The numbers whose squares are odd are:

$$\boxed{141 \text{ and } 1419}$$

7. If $87^2 = 7569$, find the value of 88^2.

We know:

$$88 = 87 + 1$$

Using the identity:

$$(a+1)^2 = a^2 + 2a + 1$$
$$88^2 = 87^2 + 2(87) + 1$$

Substitute $87^2 = 7569$:

$$88^2 = 7569 + 174 + 1$$
$$88^2 = 7743 + 1$$
$$88^2 = 7744$$

Answer: $\boxed{7744}$

---

8. Without adding, find the sum

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$$

These are the first 8 odd numbers.

Property:

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

Here $n = 8$

$$\text{Sum} = 8^2$$
$$= 64$$

Answer: $\boxed{64}$

---

9. Express 25 as the sum of 5 odd numbers.

The first 5 odd numbers are:

$$1, 3, 5, 7, 9$$

Adding them:

$$1 + 3 + 5 + 7 + 9 = 25$$

Therefore,

$$\boxed{25 = 1 + 3 + 5 + 7 + 9}$$

---

10. How many natural numbers lie between 10^2 and 11^2?

First find the squares:

$$10^2 = 100$$
$$11^2 = 121$$

Natural numbers between 100 and 121 are:

$$101, 102, 103, \ldots, 120$$

Number of terms:

$$120 - 101 + 1$$
$$= 20$$

Hence,

$$\boxed{20}$$

---

11. Find the square root by repeated subtraction method

(i) $\sqrt{121}$

Subtract consecutive odd numbers until 0 is obtained.

Step	Subtraction	Remainder
1	121−1	120
2	120−3	117
3	117−5	112
4	112−7	105
5	105−9	96
6	96−11	85
7	85−13	72
8	72−15	57
9	57−17	40
10	40−19	21
11	21−21	0

Zero is obtained after **11 subtractions**.

Therefore,

$$\boxed{\sqrt{121} = 11}$$

(ii) $\sqrt{\mathbf{144}}$

Step	Subtraction	Remainder
1	144−1	143
2	143−3	140
3	140−5	135
4	135−7	128
5	128−9	119
6	119−11	108
7	108−13	95
8	95−15	80
9	80−17	63
10	63−19	44
11	44−21	23
12	23−23	0

Zero is obtained after **12 subtractions**.

Therefore,

$$\boxed{\sqrt{144} = 12}$$

---

12. Find the square root of the following numbers

(i) $\sqrt{\mathbf{1225}}$

Prime factorisation:

$$1225 \div 5 = 245$$
$$245 \div 5 = 49$$
$$49 \div 7 = 7$$
$$7 \div 7 = 1$$

So,

$$1225 = 5 \times 5 \times 7 \times 7$$
$$= (5 \times 5)(7 \times 7)$$

Taking one factor from each pair:

$$\sqrt{1225} = 5 \times 7$$
$$= 35$$
$$\boxed{\sqrt{1225} = 35}$$

(ii) $\sqrt{2116}$

Prime factorisation:

$$2116 \div 2 = 1058$$
$$1058 \div 2 = 529$$
$$529 \div 23 = 23$$
$$23 \div 23 = 1$$

Thus,

$$2116 = 2 \times 2 \times 23 \times 23$$
$$= (2 \times 2)(23 \times 23)$$
$$\sqrt{2116} = 2 \times 23$$
$$= 46$$
$$\boxed{\sqrt{2116} = 46}$$

(iii) $\sqrt{841}$

Prime factorisation:

$$841 \div 29 = 29$$
$$29 \div 29 = 1$$

Therefore,

$$841 = 29 \times 29$$
$$\sqrt{841} = 29$$
$$\boxed{29}$$

---

13. Find the least natural number by which each number should be multiplied to get a perfect square

(i) 294

Prime factorisation:

$$294 = 2 \times 147$$
$$= 2 \times 3 \times 49$$
$$= 2 \times 3 \times 7 \times 7$$
$$= 2 \times 3 \times 7^2$$

Unpaired factors = 2and 3

Multiply by:

$$2 \times 3 = 6$$

Check:

$$294 \times 6 = 2^2 \times 3^2 \times 7^2$$

Perfect square.

$$\boxed{6}$$

(ii) 52

$$52 = 2 \times 26$$
$$= 2 \times 2 \times 13$$
$$= 2^2 \times 13$$

Unpaired factor = 13

Multiply by 13.

$$52 \times 13 = 676$$
$$676 = 26^2$$
$$\boxed{13}$$

(iii) 405

$$405 = 5 \times 81$$
$$= 5 \times 3^4$$

Unpaired factor = 5

Multiply by 5.

$$405 \times 5 = 2025$$
$$2025 = 45^2$$
$$\boxed{5}$$

(iv) 2187

Prime factorisation:

$$2187 = 3^7$$
$$= 3^6 \times 3$$

One factor of 3 remains unpaired.

Multiply by 3.

$$2187 \times 3 = 3^8$$
$$= 6561$$
$$= 81^2$$

Therefore,

$$\boxed{3}$$

Answers for Q13

$$\boxed{(i)\ 6(ii)\ 13(iii)\ 5(iv)\ 3}$$

Practice Time 1.2

1. Find the cube of the following numbers.

(i) 17

Solution By Steps

Step 1: Write the cube of 17.

$$17^3$$

$$17^3 = 17 \times 17 \times 17$$

Step 2: Multiply.

$$17 \times 17 = 289$$
$$289 \times 17 = 4913$$

Final Answer

$$\boxed{17^3 = 4913}$$

(ii) 19

Solution By Steps

$$19^3 = 19 \times 19 \times 19$$
$$19 \times 19 = 361$$
$$361 \times 19 = 6859$$

Final Answer

$$\boxed{19^3 = 6859}$$

(iii) 10

Solution By Steps

$$10^3 = 10 \times 10 \times 10$$
$$= 1000$$

Final Answer

$$\boxed{10^3 = 1000}$$

(iv) 23

Solution By Steps

$$23^3 = 23 \times 23 \times 23$$
$$23 \times 23 = 529$$
$$529 \times 23 = 12167$$

Final Answer

$$\boxed{23^3 = 12167}$$

(v) 15

Solution By Steps

$$15^3 = 15 \times 15 \times 15$$
$$15 \times 15 = 225$$
$$225 \times 15 = 3375$$

Final Answer

$$\boxed{15^3 = 3375}$$

(vi) 13

Solution By Steps

$$13^3 = 13 \times 13 \times 13$$
$$13 \times 13 = 169$$
$$169 \times 13 = 2197$$

Final Answer

$$\boxed{13^3 = 2197}$$

(vii) 25

Solution By Steps

$$25^3 = 25 \times 25 \times 25$$
$$25 \times 25 = 625$$
$$625 \times 25 = 15625$$

Final Answer

$$\boxed{25^3 = 15625}$$

(viii) 14

Solution By Steps

$$14^3 = 14 \times 14 \times 14$$
$$14 \times 14 = 196$$
$$196 \times 14 = 2744$$

Final Answer

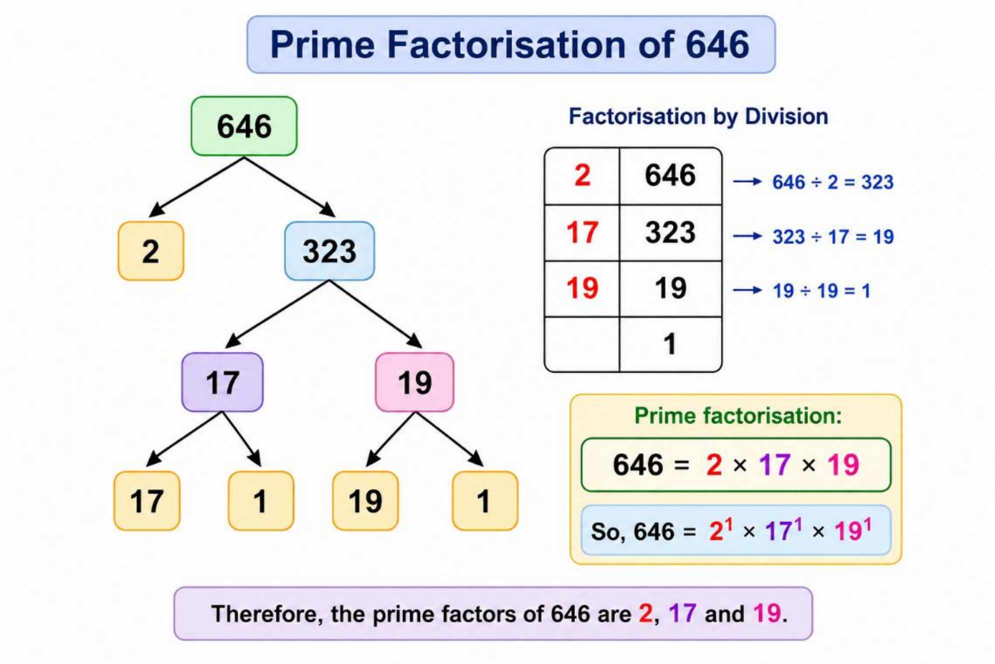
$$\boxed{14^3 = 2744}$$

---

2. Is 646 a perfect cube?

Solution By Steps

Step 1: Find the prime factorisation.



Step 2: Check groups of three.

For a perfect cube, all prime factors must occur in groups of 3.

Here,

$$646 = 2^1 \times 17^1 \times 19^1$$

No factor occurs three times.

Final Answer

$$\boxed{\text{646 is not a perfect cube.}}$$

---

3. Find the unit digit of the cube of the following numbers.

(i) 1109

Solution By Steps

Unit digit = 9

$$9^3 = 729$$

Unit digit = 9

Final Answer

$$\boxed{9}$$

(ii) 117

Unit digit = 7

$$7^3 = 343$$

Unit digit = 3

Final Answer

$$\boxed{3}$$

(iii) 565

Unit digit = 5

$$5^3 = 125$$

Unit digit = 5

Final Answer

$$\boxed{5}$$

(iv) 390

Unit digit = 0

$$0^3 = 0$$

Final Answer

$$\boxed{0}$$

(v) 662

Unit digit = 2

$$2^3 = 8$$

Final Answer

$$\boxed{8}$$

---

4. Express the following numbers as a sum of consecutive odd numbers.

(i) 6^3

Solution By Steps

$$6^3 = 216$$

A cube can be expressed as the sum of n consecutive odd numbers beginning with $n^2 - n + 1$.

For $n = 6$:

First odd number

$$6^2 - 6 + 1 = 31$$

Six consecutive odd numbers:

$$31,33,35,37,39,41$$

Checking:

$$31 + 33 + 35 + 37 + 39 + 41 = 216$$

Final Answer

$$\boxed{216 = 31 + 33 + 35 + 37 + 39 + 41}$$

(ii) 7^3

Solution By Steps

First odd number:

$$7^2 - 7 + 1 = 43$$

Seven consecutive odd numbers:

$$43 + 45 + 47 + 49 + 51 + 53 + 55$$
$$= 343$$

Final Answer

$$\boxed{343 = 43 + 45 + 47 + 49 + 51 + 53 + 55}$$

5. Find

(i) $13^3 - 12^3$

Solution By Steps

Step 1: Find the cubes.

$$13^3 = 2197$$
$$12^3 = 1728$$

Step 2: Subtract.

$$2197 - 1728 = 469$$

Final Answer

$$\boxed{13^3 - 12^3 = 469}$$

(ii) $\mathbf{10^3 - 9^3}$

Solution By Steps

$$10^3 = 1000$$
$$9^3 = 729$$
$$1000 - 729 = 271$$

Final Answer

$$\boxed{271}$$

(iii) $\mathbf{31^3 - 30^3}$

Solution By Steps

$$31^3 = 29791$$
$$30^3 = 27000$$
$$29791 - 27000 = 2791$$

Final Answer

$$\boxed{2791}$$

(iv) $\mathbf{21^3 - 20^3}$

Solution By Steps

$$21^3 = 9261$$
$$20^3 = 8000$$
$$9261 - 8000 = 1261$$

Final Answer

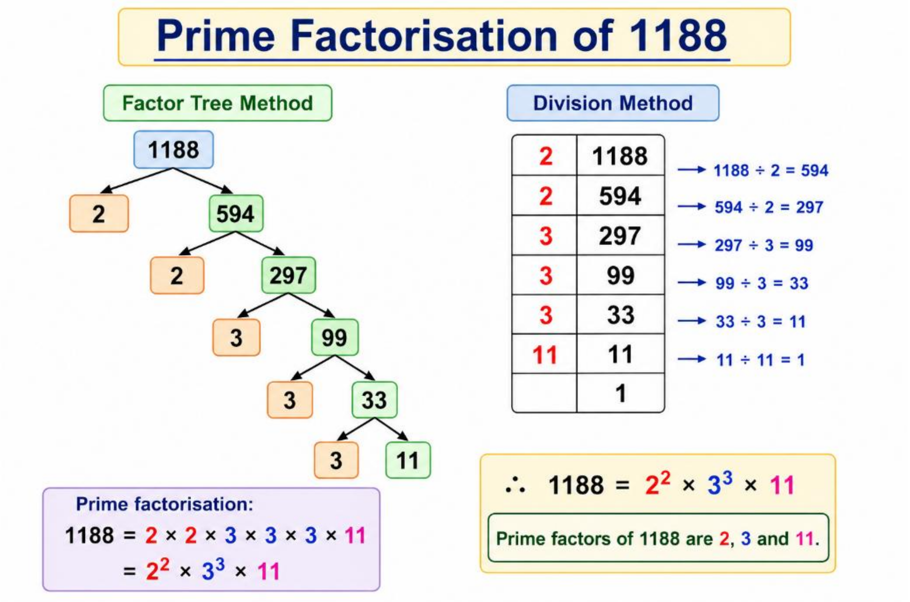
$\boxed{1261}$

---

6. Is 1188 a perfect cube? If not, find the smallest number by which 1188 must be multiplied to get a perfect cube.

Solution By Steps

Step 1: Find the prime factorisation.



Step 2: Check groups of three.

For a perfect cube, powers of all prime factors must be multiples of 3.

$$2^2 \text{(needs one more 2)}$$
$$3^3 \text{(already complete)}$$
$$11^1 \text{(needs two more 11)}$$

Step 3: Find the required multiplier.

$$2 \times 11^2$$
$$= 2 \times 121$$
$$= 242$$

Step 4: Verify.

$$1188 \times 242$$
$$= 2^3 \times 3^3 \times 11^3$$
$$= (2 \times 3 \times 11)^3$$
$$= 66^3$$

Thus the product is a perfect cube.

Final Answer

$$\boxed{\text{1188 is not a perfect cube.}}$$

The smallest number required is

$$\boxed{242}$$

7. Evaluate $\sqrt[3]{1331} + \sqrt[3]{9261}$

Solution By Steps

Step 1: Find the cube root of 1331.

$$1331 = 11^3$$
$$\sqrt[3]{1331} = 11$$

Step 2: Find the cube root of 9261.

$$9261 = 21^3$$
$$\sqrt[3]{9261} = 21$$

Step 3: Add the values.

$$11 + 21 = 32$$

Final Answer

$$\boxed{32}$$

EXAM TIME

A. Multiple Choice Questions

2. Which of the following cannot be a perfect square?

Options:

(a) 841 (b) 529 (c) 198 (d) All of these

Solution By Steps

Step 1: Check 841

$$29^2 = 841$$

841 is a perfect square.

Step 2: Check 529

$$23^2 = 529$$

529 is a perfect square.

Step 3: Check 198

Perfect squares can have units digits only:

$$0, 1, 4, 5, 6, 9$$

198 ends in 8.

A perfect square can never end in 8.

Final Answer

$$\boxed{\text{(c) } 198}$$

---

3. A number ending in 9 will have the unit place of its square as

Options:

(a) 1 (b) 3 (c) 9 (d) 6

Solution By Steps

Consider any number ending in 9.

$$9^2 = 81$$

Unit digit = 1

$$19^2 = 361$$

Unit digit = 1

$$29^2 = 841$$

Unit digit = 1

Final Answer

$$\boxed{\text{(a) 1}}$$

---

4. Which of the following is not a unit digit of a square number?

Options:

(a) 0 (b) 4 (c) 6 (d) 8

Solution By Steps

The possible unit digits of perfect squares are:

$$0, 1, 4, 5, 6, 9$$

8 is not included in the list.

Final Answer

$$\boxed{\text{(d) 8}}$$

---

5. A perfect square number having $m$digits, where $m$is even, will have square root with

Options:

(a) $m/2$digits

(b) $(m + 1)/2$digits

(c) $m + 1$digits

(d) $m/3$digits

Solution By Steps

Rule:

If a perfect square has an even number of digits, its square root contains half as many digits.

Examples:

$$144$$

has 3 digits and root 12 has 2 digits.

$$2025$$

has 4 digits and root 45 has 2 digits.

$$998001$$

has 6 digits and root 999 has 3 digits.

Thus for an even number m,

Square root has

$$\frac{m}{2}$$

digits.

Final Answer

$$\boxed{\text{(a) } \frac{m}{2}}$$

---

6. Which of the following is a square of an even number?

Options:

(a) 144 (b) 169 (c) 441 (d) 625

Solution By Steps

Check each number.

(a) 144

$$12^2 = 144$$

12 is even.

Therefore 144 is the square of an even number.

(b) 169

$$13^2 = 169$$

13 is odd.

(c) 441

$$21^2 = 441$$

21 is odd.

(d) 625

$$25^2 = 625$$

25 is odd.

Final Answer

$$\boxed{\text{(a) } 144}$$

7. How many numbers lie between the squares of 13 and 14?

Options:

(a) 28 (b) 26 (c) 13 (d) 14

Solution By Steps

Step 1: Find the squares.

$$13^2 = 169$$
$$14^2 = 196$$

Step 2: Find numbers between them.

$$196 - 169 - 1 = 26$$

Final Answer

$$\boxed{\text{(b) } 26}$$

---

8. If $45^2 = 2025$, then $46^2$is equal to

Options:

(a) 2025+46

(b) $2025+46^2$

(c) $2025+23^2$

(d) 2025+91

Solution By Steps

Using:

$$(n+1)^2 = n^2 + 2n + 1$$
$$46^2 = 45^2 + 2(45) + 1$$
$$= 2025 + 90 + 1$$
$$= 2025 + 91$$

Final Answer

$$\boxed{\text{(d) } 2025 + 91}$$

---

9. The value of $1 + 3 + 5 + 7 + 9 + 11 + 13$ is

Options:

(a) 36 (b) 41 (c) 49 (d) 52

Solution By Steps

There are 7 odd numbers.

Sum of first n odd numbers:

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$
$$= 7^2$$
$$= 49$$

Final Answer

$$\boxed{\text{(c) } 49}$$

---

10. The value of $198^2 - 197^2$ is

Options:

(a) 393 (b) 395 (c) 295 (d) 293

Solution By Steps

Using:

$$a^2 - b^2 = (a - b)(a + b)$$
$$= (198 - 197)(198 + 197)$$
$$= 1 \times 395$$
$$= 395$$

Final Answer

$$\boxed{\text{(b) } 395}$$

---

11. The square root of 196 is

Options:

(a) 12 (b) 11 (c) 15 (d) 14

Solution By Steps

$$14^2 = 196$$

Therefore,

$$\sqrt{196} = 14$$

Final Answer

$$\boxed{\text{(d) } 14}$$

---

12. The smallest square number divisible by 2, 3 and 8 is

Options:

(a) 64 (b) 144 (c) 1296 (d) 576

Solution By Steps

LCM of 2, 3 and 8:

$$24 = 2^3 \times 3$$

To make a perfect square:

$$24 \times 2 \times 3 = 144$$
$$144 = 12^2$$

Final Answer

$\boxed{\text{(b) } 144}$

---

13. The cube of 24 is

Options:

(a) 12167

(b) 15625

(c) 13824

(d) 10648

Solution By Steps

$$24^3 = 24 \times 24 \times 24$$
$$= 576 \times 24$$
$$= 13824$$

Final Answer

$\boxed{\text{(c) } 13824}$

---

14. Which of the following number is a perfect cube?

Options:

(a) 243

(b) 512

(c) 392

(d) 8640

Solution By Steps

$$512 = 8^3$$

Hence 512 is a perfect cube.

Final Answer

$$\boxed{\text{(b) } 512}$$

---

15. Which of the following numbers is not a perfect cube?

Options:

(a) 216

(b) 343

(c) 125

(d) 667

Solution By Steps

$$216 = 6^3$$
$$343 = 7^3$$
$$125 = 5^3$$

667 is not a cube.

Final Answer

$$\boxed{\text{(d) } 667}$$

---

16. Which of the following is the cube of an even natural number?

Options:

(a) 1331

(b) 4913

(c) 3375

(d) 1728

Solution By Steps

$$1331 = 11^3$$
$$4913 = 17^3$$
$$3375 = 15^3$$
$$1728 = 12^3$$

Only 12 is even.

Final Answer

$$\boxed{\text{(d) } 1728}$$

---

17. The ones digit of the cube of 23 is

Options:

(a) 3

(b) 6

(c) 7

(d) 9

Solution By Steps

Unit digit of 23 = 3

$$3^3 = 27$$

Unit digit = 7

Final Answer

$$\boxed{\text{(c) } 7}$$

---

18. The value of $21 + 23 + 25 + 27 + 29$is equal to

Options:

(a) 3^3

(b) 6^3

(c) 4^3

(d) 5^3

Solution By Steps

$$21 + 23 + 25 + 27 + 29$$
$$= 125$$
$$125 = 5^3$$

Final Answer

$$\boxed{\text{(d) } 5^3}$$

---

19. The value of $23^3 - 22^3$is

Options:

(a) 1518

(b) 1519

(c) 1421

(d) 1420

Solution By Steps

$$23^3 = 12167$$
$$22^3 = 10648$$
$$12167 - 10648 = 1519$$

Final Answer

$$\boxed{\text{(b) } 1519}$$

---

20. The smallest number by which 841 must be multiplied so that the product becomes a perfect cube is

Options:

(a) 29

(b) 27

(c) 25

(d) 23

Solution By Steps

Prime factorisation:

$$841 = 29^2$$

For a perfect cube, powers must be multiples of 3.

Multiply by one more 29.

$$29^2 \times 29 = 29^3$$

Final Answer

$$\boxed{\text{(a) } 29}$$

---

21. The cube root of 6859 is

Options:

(a) 19

(b) 18

(c) 17

(d) 21

Solution By Steps

$$\begin{aligned} 19^3 &= 19 \times 19 \times 19 \\ &= 361 \times 19 \\ &= 6859 \end{aligned}$$

Final Answer

$$\boxed{\text{(a) } 19}$$

22. What is the value of

$$\frac{\sqrt[3]{64} + \sqrt[3]{125}}{\sqrt[3]{27}}$$

Options:

(a) 2

(b) 3

(c) 4

(d) 9

Solution By Steps

$$\sqrt[3]{64} = 4$$
$$\sqrt[3]{125} = 5$$
$$\sqrt[3]{27} = 3$$

Substituting:

$$\frac{4+5}{3}$$
$$= \frac{9}{3}$$
$$= 3$$

Final Answer

$$\boxed{\text{(b) 3}}$$

B. Fill in the Blanks

1. The pair/pairs of partner factors of 5 is/are _________.

Solution By Steps

Step 1: Find the factors of 5.

Factors of 5 are:

$$1, 5$$

Step 2: Form factor pairs.

$$1 \times 5 = 5$$

Thus the partner factor pair is:

$$(1,5)$$

Final Answer

$$\boxed{(1,5)}$$

---

2. The unit digit in the square of 1456 is _________.

Solution By Steps

Step 1: Observe the unit digit of the number.

1456 ends in 6.

Step 2: Find the unit digit of its square.

$$6^2 = 36$$

Unit digit = 6

Final Answer

$$\boxed{6}$$

---

3. The 26th odd number is _________.

Solution By Steps

The n^{th}odd number is:

$$2n - 1$$

For $n = 26$,

$$\begin{aligned}&2(26) - 1\\&52 - 1\\&51\end{aligned}$$

Final Answer

$$\boxed{51}$$

---

4. The square root of 24025 will have _________ digits.

Solution By Steps

24025 has 5 digits.

Grouping from right:

$$2 \ | \ 40 \ | \ 25$$

There are 3 groups.

The square root contains as many digits as groups.

Final Answer

$$\boxed{3}$$

---

5. The least number by which 125 must be multiplied to make it a perfect square is _________.

Solution By Steps

Prime factorisation:

$$125 = 5 \times 5 \times 5$$
$$= 5^3$$

For a perfect square, powers must be even.

Multiply by one more 5:

$$5^3 \times 5 = 5^4$$
$$= 625$$

which is a perfect square.

Final Answer

$$\boxed{5}$$

---

6. The cube of 12.5 is _________.

Solution By Steps

$$12.5^3$$
$$= 12.5 \times 12.5 \times 12.5$$
$$= 156.25 \times 12.5$$
$$= 1953.125$$

Final Answer

$$\boxed{1953.125}$$

---

7. Cube of a number ending in 7 will end in the digit _________.

Solution By Steps

$$7^3 = 343$$

Unit digit = 3

Similarly,

$$17^3, 27^3, 37^3$$

all end in 3.

Final Answer

$$\boxed{3}$$

---

8. The cube of 90 will have __________ zeros.

Solution By Steps

$$\begin{aligned}
&90 = 9 \times 10 \\
&90^3 = 9^3 \times 10^3 \\
&= 729 \times 1000 \\
&= 729000
\end{aligned}$$

There are 3 zeros.

Final Answer

$$\boxed{3}$$

---

9. The value of $7^3 - 6^3$is __________.

Solution By Steps

$$\begin{aligned}
7^3 &= 343 \\
6^3 &= 216
\end{aligned}$$

$$343 - 216$$
$$= 127$$

Final Answer

$$\boxed{127}$$

---

10. The least number by which 72 is divided to make it a perfect cube is __________.

Solution By Steps

Prime factorisation:

$$72 = 2^3 \times 3^2$$

For a perfect cube, powers must be multiples of 3.

Remove $3^2$by dividing by 9:

$$72 \div 9 = 8$$
$$8 = 2^3$$

which is a perfect cube.

Final Answer

$$\boxed{9}$$

C. True / False

---

1. The square of 2.3 is 5.29.

Solution By Steps

Step 1: Find the square of 2.3.

$$2.3^2 = 2.3 \times 2.3$$

$$= \frac{23}{10} \times \frac{23}{10}$$
$$= \frac{529}{100}$$
$$= 5.29$$

Final Answer

$$\boxed{\text{True}}$$

---

2. The square of 87 will have 3 at the unit place.

Solution By Steps

Step 1: Look at the unit digit.

87 ends in 7.

Step 2: Find the unit digit of its square.

$$7^2 = 49$$

Unit digit = 9

Final Answer

$$\boxed{\text{False}}$$

The unit digit is 9, not 3.

---

3. The number of zeroes at the end of a perfect square is always even.

Solution By Steps

A perfect square has pairs of factors.

Examples:

$$10^2 = 100$$

(2 zeros)

$$100^2 = 10000$$

(4 zeros)

$$1000^2 = 1000000$$

(6 zeros)

Thus trailing zeros always occur in even numbers.

Final Answer

$$\boxed{\text{True}}$$

---

4. The sum of first 36 odd natural numbers is 1260.

Solution By Steps

Sum of first $n$odd numbers:

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

For $n = 36$,

$$36^2 = 1296$$

Given value:

$$1260$$
$$1296 \neq 1260$$

Final Answer

$$\boxed{\text{False}}$$

Correct sum:

$$\boxed{1296}$$

---

5. The cube of 1.5 is 3.325.

Solution By Steps

$$\begin{aligned}&1.5^3\\&= 1.5 \times 1.5 \times 1.5\\&= 2.25 \times 1.5\\&= 3.375\end{aligned}$$

Given value:

$$\begin{aligned}&3.325\\&3.375 \neq 3.325\end{aligned}$$

Final Answer

$$\boxed{\text{False}}$$

---

6. 999 is a perfect cube.

Solution By Steps

Nearby cubes are:

$$\begin{aligned}&9^3 = 729\\&10^3 = 1000\end{aligned}$$

Since

$$999 \neq 729$$

and

$$999 \neq 1000$$

999 is not a perfect cube.

Final Answer

$$\boxed{\text{False}}$$

---

7. Cube roots of 8 are +2 and –2.

Solution By Steps

$$2^3 = 8$$

but

$$(-2)^3 = -8$$

Cube root of 8 is only:

$$\sqrt[3]{8} = 2$$

Final Answer

$$\boxed{\text{False}}$$

D. Match the Columns

---

1. Match the Column I with Column II

Column I (Number)

(a) 441

(b) 289

(c) 81

(d) 196

Column II (Square Root)

(i) 14

(ii) 9

(iii) 17

(iv) 21

Solution By Steps

(a) 441

Find the square root:

$$21^2 = 441$$

Therefore,

$$\sqrt{441} = 21$$

Matches with (iv)

(b) 289

$$17^2 = 289$$

Therefore,

$$\sqrt{289} = 17$$

Matches with (iii)

(c) 81

$$9^2 = 81$$

Therefore,

$$\sqrt{81} = 9$$

Matches with (ii)

(d) 196

$$14^2 = 196$$

Therefore,

$$\sqrt{196} = 14$$

Matches with (i)

Final Answer

Column I	Column II
(a) 441	(iv) 21
(b) 289	(iii) 17
(c) 81	(ii) 9
(d) 196	(i) 14

---

2. Match the Column I with Column II

Column I

(a) Cube of 26

(b) Cube root of 21952

(c) $\sqrt[3]{125} + (\sqrt[3]{625})^3$

(d) $(\sqrt[3]{25})^3 + \sqrt[3]{6859}$

Column II

(i) 17576

(ii) 144

(iii) 28

(iv) 15630

Solution By Steps

(a) Cube of 26

$$26^3 = 26 \times 26 \times 26$$
$$= 676 \times 26$$
$$= 17576$$

Matches with (i)

(b) Cube root of 21952

$$28^3 = 21952$$

Therefore,

$$\sqrt[3]{21952} = 28$$

Matches with (iii)

(c) $\sqrt[3]{\mathbf{125}} + (\sqrt{\mathbf{625}})^{\mathbf{3}}$

Solution By Steps

Step 1: Evaluate the cube root.

$$\sqrt[3]{125} = 5$$

Step 2: Evaluate the square root.

$$\sqrt{625} = 25$$

Step 3: Cube the result.

$$25^3 = 15625$$

Step 4: Add.

$$5 + 15625 = 15630$$

Final Answer

$$\boxed{15630}$$

(d) $\left(\sqrt{25}\right)^3 + \sqrt[3]{6859}$

Solution By Steps

Step 1: Evaluate the square root.

$$\sqrt{25} = 5$$

Step 2: Cube the result.

$$5^3 = 125$$

Step 3: Evaluate the cube root.

$$\sqrt[3]{6859} = 19$$

because

$$19^3 = 6859.$$

Step 4: Add.

$$125 + 19 = 144$$

Final Answer

$$\boxed{144}$$

Column I Column II

(a) (i)

Column I	Column II
(b)	(iii)
(c)	(iv)
(d)	(ii)

E. Very Short Answer Type Questions

1. Find the pairs of partner factors of 15.

Solution By Steps

Factor pairs of 15 are:

$$1 \times 15 = 15$$
$$3 \times 5 = 15$$

Final Answer

$$\boxed{(1,15) \text{ and } (3,5)}$$

---

2. Write the first five square numbers.

Solution By Steps

$$1^2 = 1$$
$$2^2 = 4$$
$$3^2 = 9$$
$$4^2 = 16$$
$$5^2 = 25$$

Final Answer

$$\boxed{1, 4, 9, 16, 25}$$

E. Very Short Answer Type Questions

---

3. Write all perfect square numbers between 50 and 100.

Solution By Steps

Step 1: Find square numbers near 50 and 100.

$$7^2 = 49$$
$$8^2 = 64$$
$$9^2 = 81$$
$$10^2 = 100$$

Step 2: Select those lying strictly between 50 and 100.

$$64, 81$$

Final Answer

$$\boxed{64, 81}$$

---

4. Find the unit digit of the square of 2379.

Solution By Steps

Step 1: Observe the unit digit.

2379 ends in 9.

Step 2: Square the unit digit.

$$9^2 = 81$$

Unit digit = 1.

Final Answer

$$\boxed{1}$$

---

5. Find the square root of 324.

Solution By Steps

$$18 \times 18 = 324$$

Therefore,

$$\sqrt{324} = 18$$

Final Answer

$$\boxed{18}$$

---

6. Is 2025 a perfect square?

Solution By Steps

$$45 \times 45 = 2025$$

or

$$2025 = 3^4 \times 5^2$$

All prime factors occur in pairs.

Final Answer

$$\boxed{\text{Yes, 2025 is a perfect square.}}$$

---

7. Find the cube of 18.

Solution By Steps

$$\begin{aligned} 18^3 &= 18 \times 18 \times 18 \\ &= 324 \times 18 \\ &= 5832 \end{aligned}$$

Final Answer

$$\boxed{5832}$$

---

8. Find the cube root of 2744.

Solution By Steps

$$14^3 = 14 \times 14 \times 14$$
$$= 196 \times 14$$
$$= 2744$$

Therefore,

$$\sqrt[3]{2744} = 14$$

Final Answer

$$\boxed{14}$$

---

9. What is the unit digit of the cube of 32?

Solution By Steps

Step 1: Observe the unit digit.

32 ends in 2.

Step 2: Find cube of 2.

$$2^3 = 8$$

Therefore, the unit digit of $32^3$is 8.

Final Answer

$$\boxed{8}$$

---

10. Is 512 a perfect cube?

Solution By Steps

$$8^3 = 512$$

Hence, 512 is a perfect cube.

Final Answer

$$\boxed{\text{Yes}}$$

---

11. Find the value of $11^3 - 10^3$.

Solution By Steps

$$11^3 = 1331$$
$$10^3 = 1000$$
$$1331 - 1000 = 331$$

Final Answer

$$\boxed{331}$$

---

12. Find the smallest number by which 54 must be multiplied to get a perfect cube.

Solution By Steps

Step 1: Prime factorisation of 54.

$$54 = 2 \times 3 \times 3 \times 3$$
$$= 2 \times 3^3$$

Step 2: Check groups of three.

$3^3$is complete.

Factor 2 needs two more 2's.

Multiply by:

$$2^2 = 4$$

Step 3: Verify.

$$54 \times 4$$
$$= 2^3 \times 3^3$$
$$= (2 \times 3)^3$$
$$= 6^3$$

Final Answer

$$\boxed{4}$$

F. Short Answer Type Questions

---

1. Find the partner factors of 36.

Solution By Steps

Step 1: Find the factors of 36.

$$1,\ 2,\ 3,\ 4,\ 6,\ 9,\ 12,\ 18,\ 36$$

Step 2: Form factor pairs.

$$1 \times 36 = 36$$
$$2 \times 18 = 36$$
$$3 \times 12 = 36$$
$$4 \times 9 = 36$$
$$6 \times 6 = 36$$

Final Answer

$$\boxed{(1,36),\ (2,18),\ (3,12),\ (4,9),\ (6,6)}$$

---

2. Check whether 784 is a perfect square or not.

Solution By Steps

Step 1: Find the prime factorisation.

2	784
2	392
2	196
2	98
7	49
7	7
	1

Therefore,

$$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$
$$= 2^4 \times 7^2$$

Step 2: Make pairs.

$$784 = (2 \times 2)(2 \times 2)(7 \times 7)$$

All factors occur in pairs.

Step 3: Find square root.

$$\sqrt{784} = 2 \times 2 \times 7$$
$$= 28$$

Final Answer

$$\boxed{\text{784 is a perfect square.}}$$
$$\boxed{\sqrt{784} = 28}$$

---

3. Find the square root of 1849.

Solution By Steps

Step 1: Prime factorisation

$$\begin{array}{r|r} 43 & 1849 \\ 43 & 43 \\ & 1 \end{array}$$

Thus,

$$1849 = 43 \times 43$$

Step 2: Take one factor from each pair.

$$\sqrt{1849} = 43$$

Final Answer

$$\boxed{43}$$

---

4. Find the cube of 35.

Solution By Steps

$$\begin{aligned} 35^3 &= 35 \times 35 \times 35 \\ &= 1225 \times 35 \\ &= 42875 \end{aligned}$$

Final Answer

$$\boxed{42875}$$

---

5. Check whether 3375 is a perfect cube.

Solution By Steps

Step 1: Prime factorisation

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Therefore,

$$3375 = 3^3 \times 5^3$$

Step 2: Check groups of three.

All prime factors occur in groups of three.

Step 3: Find cube root.

$$\sqrt[3]{3375}$$
$$= 3 \times 5$$
$$= 15$$

Final Answer

$$\boxed{\text{3375 is a perfect cube.}}$$
$$\boxed{\sqrt[3]{3375} = 15}$$

---

6. Find the cube root of 13824.

Solution By Steps

Step 1: Prime factorisation

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Therefore,

$$13824 = 2^9 \times 3^3$$

Step 2: Make groups of three.

$$= (2^3)(2^3)(2^3)(3^3)$$

Step 3: Take one factor from each group.

$$\sqrt[3]{13824}$$
$$= 2 \times 2 \times 2 \times 3$$
$$= 24$$

Final Answer

$$\boxed{24}$$

---

7. Find the value of $15^2 - 14^2$.

Solution By Steps

Using:

$$a^2 - b^2 = (a - b)(a + b)$$

$$a$$

$$b$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$9^2 - 4^2 = (9 - 4)(9 + 4) = 65$$

aba + ba - b

$$\begin{aligned} & 15^2 - 14^2 \\ & = (15 - 14)(15 + 14) \\ & = 1 \times 29 \\ & = 29 \end{aligned}$$

Final Answer

$$\boxed{29}$$

---

8. Find the value of $18^3 - 17^3$.

Solution By Steps

$$\begin{aligned} & 18^3 = 5832 \\ & 17^3 = 4913 \\ & 5832 - 4913 \\ & = 919 \end{aligned}$$

Final Answer

$$\boxed{919}$$

G. Long Answer Type Questions

---

1. Find the square root of 24025 by the Prime Factorisation Method.

Solution By Steps

Step 1: Find the prime factorisation of 24025.

5	24025
5	4805
31	961
31	31
	1

Therefore,

$$24025 = 5 \times 5 \times 31 \times 31$$
$$= 5^2 \times 31^2$$

Step 2: Make pairs of equal factors.

$$24025 = (5 \times 5)(31 \times 31)$$

Step 3: Take one factor from each pair.

$$\sqrt{24025}$$
$$= 5 \times 31$$
$$= 155$$

Final Answer

$$\boxed{\sqrt{24025} = 155}$$

---

2. Check whether 2916 is a perfect square. If yes, find its square root.

Solution By Steps

Step 1: Prime factorisation

2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Therefore,

$$2916 = 2^2 \times 3^6$$

Step 2: Make pairs

$$2916 = (2 \times 2)(3 \times 3)(3 \times 3)(3 \times 3)$$

All factors occur in pairs.

Step 3: Find square root

$$\sqrt{2916}$$
$$= 2 \times 3 \times 3 \times 3$$
$$= 54$$

Final Answer

$$\boxed{\text{2916 is a perfect square.}}$$
$$\boxed{\sqrt{2916} = 54}$$

---

3. Find the smallest number by which 432 must be multiplied so that the product becomes a perfect cube.

Solution By Steps

Step 1: Prime factorisation of 432

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Therefore,

$$432 = 2^4 \times 3^3$$

Step 2: Check groups of three

For a perfect cube, powers must be multiples of 3.

$$2^4 = 2^3 \times 2$$

One factor 2 remains.

$$3^3$$

is already complete.

Step 3: Multiply by the required factor

Multiply by

$$2^2 = 4$$

Then,

$$\begin{aligned} &432 \times 4 \\ &= 2^6 \times 3^3 \\ &= (2^2 \times 3)^3 \\ &= 12^3 \end{aligned}$$

Final Answer

$$\boxed{4}$$

---

4. Find the cube root of 46656 by Prime Factorisation Method.

Solution By Steps

Step 1: Prime factorisation

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Therefore,

$$46656 = 2^6 \times 3^6$$

Step 2: Make groups of three

$$= (2^3)(2^3)(3^3)(3^3)$$

Step 3: Take one factor from each group

$$\sqrt[3]{46656}$$
$$= 2 \times 2 \times 3 \times 3$$

$$= 36$$

Final Answer

$$\boxed{\sqrt[3]{46656} = 36}$$

---

5. Find the value of

$$25^2 - 24^2$$

Solution By Steps

Using:

$$a^2 - b^2 = (a - b)(a + b)$$

$$a$$

$$b$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$9^2 - 4^2 = (9 - 4)(9 + 4) = 65$$

aba + ba - b

$$\begin{aligned} &25^2 - 24^2 \\ &= (25 - 24)(25 + 24) \\ &= 1 \times 49 \\ &= 49 \end{aligned}$$

Final Answer

$$\boxed{49}$$

---

6. Find the value of

$$31^3 - 30^3$$

Solution By Steps

$$31^3 = 29791$$
$$30^3 = 27000$$
$$29791 - 27000$$
$$= 2791$$

Final Answer

$$\boxed{2791}$$

Competency-Based

A. Assertion–Reason Questions

1.

- Assertion: The square root of a perfect square is always a whole number. ✔
- Reason: A perfect square can be expressed as the product of an integer by itself. ✔
- The reason correctly explains the assertion.

Answer: (a)

2.

- Assertion: 6838 is a perfect square number. ✖
  - A perfect square cannot end in 2, 3, 7, or 8.
- Reason: A perfect square number can never have 8 at the unit place. ✔

Answer: (d)

3.

- Assertion: The cube of an even number is always an even number. ✔
- Reason: Multiplying an even number by itself three times always gives an even number. ✔
- The reason correctly explains the assertion.

Answer: (a)

4.

- Assertion: Finding the cube root of a number is the inverse operation of finding the cube of that number. ✔
- Reason: If you take the cube root of a number and then cube it, you get back the original number. ✔
- The reason correctly explains the assertion.

Answer: (a)

B. Case Study Question (i)

Find the cube roots of **13824** and **175616**.

$$24^3 = 24 \times 24 \times 24 = 13824$$

Therefore,

$$\sqrt[3]{13824} = 24$$

Also,

$$56^3 = 56 \times 56 \times 56 = 175616$$

Therefore,

$$\sqrt[3]{175616} = 56$$

Answer: (b) 24, 56

Maths Booster Magic Square

Given:

$$\sqrt{225} = 15, \sqrt{144} = 12, \sqrt{81} = 9$$

So the square becomes:

$$\begin{array}{|ccc|} \hline 15 & ? & 13 \\ ? & 12 & ? \\ ? & ? & 9 \\ \hline \end{array}$$

The diagonal sum is:

$$15 + 12 + 9 = 36$$

Hence the magic sum is **36**.

Using row, column, and diagonal sums:

$$\begin{aligned} &15 + b + 13 = 36 \Rightarrow b = 8 \\ &13 + f + 9 = 36 \Rightarrow f = 14 \\ &12 + 14 + h = 36 \Rightarrow h = 10 \\ &15 + d + e = 36 \Rightarrow d + e = 21 \\ &e + 10 + 9 = 36 \Rightarrow e = 17 \\ &d + 17 = 21 \Rightarrow d = 4 \end{aligned}$$

Completed square:

$$\begin{array}{|ccc|} \hline 15 & 8 & 13 \\ 4 & 12 & 14 \\ 17 & 10 & 9 \\ \hline \end{array}$$

Chapter 2 Power Play

NCERT CORNER

Intext Questions

Question 1.
Which expression describes the thickness of a sheet of paper after it is folded 10 times? The initial thickness is represented by the letter-number v.
(i) 10v
(ii) 10 + v
(iii) $2 \times 10 \times v$
(iv) 2^{10}
(v) 2^{10} v
(vi) 10^2 v
Solution:
Each time a sheet of paper is folded, its thickness doubles. So after:
1 fold → thickness = $2 \times v$
2 folds → thickness = $2^2 \times v$
Similarly, 10 folds → thickness = $2^{10} \times v$
Hence, the correct option is (v) $2^{10} \times v$.
When we have a negative number like -2 and we raise it to different powers, the sign of the result depends on whether the power is even or odd.
A negative number raised to an even power gives a positive result.
$(-2)^2 = (-2) \times (-2) = 4$
A negative number raised to an odd power gives a negative result.
$(-2)^3 = (-2) \times (-2) \times (-2) = -8$.

Question 2.
What is $(-1)^5$? Is it positive or negative? What about $(-1)^{56}$?
Solution:
$(-1)^5 = -1$, which is negative. [$\because$ $(-1)^{0\text{dd number}} = -1$ → Negative]
$(-1)^{56} = +1$, which is positive. [$\because$ $(-1)^{\text{even number}} = +1$ → positive]

Question (II) .
What is 0^2, 0^5?
Answer:
$0^2 = 0 \times 0 = 0$
$0^5 = 0 \times 0 \times 0 \times 0 \times 0 = 0$

Question (iii).
What is 0^n?

Answer:
$0^n = 0 \times 0 \times 0 \times \ldots\ldots$ n times = 0

Question 3.
Is $(-2)^4 = 16$? Verify.
Solution:
$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$
$= 4 \times 4$
$= 16$
Yes, $(-2)^4 = 16$.

Figure It Out

Question 1.
Express the following in exponential form:
(i) $6 \times 6 \times 6 \times 6$
(ii) $y \times y$
(iii) $b \times b \times b \times b$
(iv) $5 \times 5 \times 7 \times 7 \times 7$
(v) $2 \times 2 \times a \times a$
(vi) $a \times a \times a \times c \times c \times c \times c \times d$
Solution:
(i) $6 \times 6 \times 6 \times 6 = 6^4$
(ii) $y \times y = y^2$
(iii) $b \times b \times b \times b = b^4$
(iv) $5 \times 5 \times 7 \times 7 \times 7 = 5^2 \times 7^3$
(v) $2 \times 2 \times a \times a = 2^2 \times a^2$
(vi) $a \times a \times a \times c \times c \times c \times c \times d = a^3 \times c^4 \times d^1$

Question 2.
Express each of the following as a product of powers of their prime factors in exponential form.
(i) 648
(ii) 405
(iii) 540
(iv) 3600
Solution:
(i) Prime factors of 648 = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
Exponential form of 648 = $2^3 \times 3^4$

(ii) Prime factors of 405 = $3 \times 3 \times 3 \times 3 \times 5$
Exponential form of 405 = $3^4 \times 5$

(iii) Prime factors of 540 = $2 \times 2 \times 3 \times 3 \times 3 \times 5$
Exponential form of 540 = $2^2 \times 3^3 \times 5$

(iv) Prime factors of 3600 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
Exponential form of 3600 = $2^4 \times 3^2 \times 5^2$

Question 3.
Write the numerical value of each of the following:
(i) 2×10^3
(ii) $7^2 \times 2^3$
(iii) 3×4^4
(iv) $(-3)^2 \times (-5)^2$
(v) $3^2 \times 10^4$
(vi) $(-2)^5 \times (-10)^6$
Solution:
(i) $2 \times 10^3 = 2 \times 10 \times 10 \times 10$
$= 2 \times 1000$
$= 2000$

(ii) $7^2 \times 2^3 = 7 \times 7 \times 2 \times 2 \times 2$
$= 49 \times 8$
$= 392$

(iii) $3 \times 4^4 = 3 \times 4 \times 4 \times 4 \times 4$
$= 3 \times 256$
$= 768$

(iv) $(-3)^2 \times (-5)^2 = -3 \times -3 \times -5 \times -5$
$= 9 \times 25$
$= 225$
Negative numbers raised to even powers become positive.

(v) $3^2 \times 10^4 = 3 \times 3 \times 10 \times 10 \times 10 \times 10$
$= 9 \times 10{,}000$
$= 90{,}000$

(vi) $(-2)^5 \times (-10)^6 = -2 \times -2 \times -2 \times -2 \times -2 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= (-32) \times (10{,}00{,}000)$
$= -3{,}20{,}00{,}000$
Odd power of a negative number remains negative, even power becomes positive.

INTEXT

Question 1.
Use this observation to compute the following. (Page 24)
(i) 2^9
(ii) 5^7
(iii) 4^6
Solution:
(i) $2^9 = 2^3 \times 2^3 \times 2^3$
$= 8 \times 8 \times 8$
$= 512$

(ii) $5^7 = 5^2 \times 5^2 \times 5^2 \times 5$
$= 25 \times 25 \times 25 \times 5$
$= 625 \times 125$
$= 78125$

(iii) $4^6 = 4^2 \times 4^2 \times 4^2$
$= 16 \times 16 \times 16$
$= 256 \times 16$
$= 4096$

Question 2.
Write the following expressions as a power of a power in at least two different ways:
(i) 8^6
(ii) 7^{15}
(iii) 9^{14}
(iv) 5^8
Solution:

Expression	Way 1	Way 2
(*i*) 8^6	$(8^2)^3$	$(8^3)^2$
(*ii*) 7^{15}	$(7^5)^3$	$(7^3)^5$
(*iii*) 9^{14}	$(9^2)^7$	$(9^7)^2$
(*iv*) 5^8	$(5^2)^4$	$(5^4)^2$

Magical Pond

Question 1.
Write the number of lotuses (in exponential form) when the pond was
(i) fully covered
(ii) half covered (Page 25)
Solution:
The number of lotuses doubles every day.
On the 30th day, the pond is fully covered.
So, on the 29th day, the pond must be half full.
Number of lotuses:
Day 1 → 1 = 2^0
Day 2 → 2^1
Day 3 → 2^2
Day 4 → 2^3
.
.
.
Day 29 → 2^{28}
Day 30 → 2^{29}
(i) The number of lotuses when the pond was fully covered (Day 30) = 2^{29}
(ii) The number of lotuses when the pond was half covered (Day 29) = 2^{28}

Question 2.
Simplify
$\frac{10^4}{5^4}$ and write it in exponential form.
Solution:
Step 1: Write the expression

$$\frac{10^4}{5^4}$$

Step 2: Use the exponent rule

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

So,

$$\frac{10^4}{5^4} = \left(\frac{10}{5}\right)^4$$

Step 3: Simplify inside the brackets

$$\left(\frac{10}{5}\right)^4 = 2^4$$

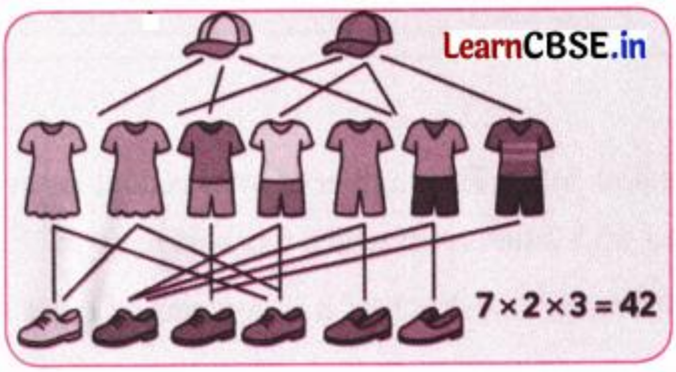
Final Answer

$$\boxed{2^4}$$

(If evaluated, $2^4 = 16$.)

IN TEXT

Question 1.
Roxie has 7 dresses, 2 hats, and 3 pairs of shoes. How many different ways can Roxie dress up?
Solution:



Roxie, pick one dress from the 7 options.
Pair it with one hat from the 2 choices.
Then choose one pair of shoes from the 3 styles.
Each piece is independent, so we multiply the choices = 7 × 2 × 3 = 42
That means Roxie has 42 unique combinations to show off!

Question 2.
Think about how many combinations are possible in different contexts. (Page 27)
(i) Pin codes of places in India
The Pincode of Vidisha in Madhya Pradesh is 464001.
The Pincode of Zemabawk in Mizoram is 796017.
(ii) Mobile numbers.
(iii) Vehicle registration numbers.
Try to find out how these numbers or codes are allotted/generated.

Solution:
(i) PIN Codes in India
India uses a 6-digit Postal Index Number (PIN) system introduced in 1972.
Structure:

- 1st digit: Region (9 zones total)
- 2nd digit: Sub-region
- 3rd digit: Sorting district
- Last 3 digits: Specific post office

Combinations:

- Theoretically: 10^6 = 1,000,000 combinations
- Practically: Only valid combinations are used based on geography and postal infrastructure.
  e.g, 464001 → Vidisha, Madhya Pradesh
  796017 → Zemabawk, Mizoram

(ii) Mobile numbers in India
Indian mobile numbers are 10 digits, starting with digits 6-9.
Structure:

- 1st digit: Must be 6, 7, 8, or 9
- Remaining 9 digits: Any number from 0-9

Combinations:
4×10^9 = 4,000,000,000 possible mobile numbers
Allocation:

- Managed by the Department of Telecommunications
- Prefixes (like 91x, 98x) are assigned to different telecom operators.

(iii) Vehicle Registration Numbers
Vehicle numbers follow a format like DL 01 AB 1234.
Structure:

- 2 letters: State/UT code (e.g., DL for Delhi)
- 2 digits: RTO code
- 1-2 letters: Series
- 4 digits: Unique vehicle number

Combinations:
Varies by state and RTO, but for one RTO:
26^2 letter combinations × 10^4 numbers = 676 × 10,000 = 6,760,000 combinations per RTO
Allocation:

- Managed by the Ministry of Road Transport & Highways via the Parivahan portal.
- Fancy numbers can be bid for, and older vehicles may require re-registration.

Question 3.
What is $2^{100} \div 2^{25}$ in powers of 2? (Page 27)
Solution:
$2^{100} \div 2^{25} = 2^{100-25} = 2^{75}$
So, dividing powers is just like reducing the number of times we multiply the base.
This rule helps us solve problems faster and understand how powers behave when we break them down.

IN TEXT

Question 1.
We had required a and b to be counting numbers? Can a and b be any integers? Will the generalised forms still hold? (Page 29)
Solution:
For powers with the same base:
$n^a \div n^b = n^{a-b}$
This rule originally uses a and b as counting numbers (i.e., 1, 2, 3,...).
But what happens if a and b are any integers, including zero or negative numbers?
Yes, the rule still holds as long as n ≠ 0.
Case 1: a and b are positive.
$\frac{2^{11}}{2^7} = 2^{11-7} = 2^4 = 16$
Case 2: a or b is zero
$\frac{2^0}{2^5} = 2^{0-5} = 2^{-5} = \frac{1}{32}$
We can write $x^{-a} = \frac{1}{x^a}$
Case 3: a and/or b are negative.
$\frac{2^{-2}}{2^{-5}} = 2^{-2+5} = 2^3 = 8$

Question 2.

(i) $2^{-4}=\frac{1}{2^4}=\frac{1}{16}$

(ii) $10^{-5}=\frac{1}{10^5}=\frac{1}{1,00,000}$

(iii) $(-7)^{-2}=\frac{1}{(-7)^2}=\frac{1}{49}$

(iv) $(-5)^{-3}=\frac{1}{(-5)^3}=\frac{1}{-125}$

(v) $10^{-100}=\frac{1}{10^{100}}$

Question3.

Solution:

(i) $2^{-4} \times 2^7$

$= \frac{1}{2^4} \times 2^7$

$= 2^{7-4}$ $\quad (\because a^m \div a^n = a^{m-n})$

$= 2^3$ LearnCBSE.in

(ii) $3^2 \times 3^{-5} \times 3^6$

$= 3^2 \times \frac{1}{3^5} \times 3^6$

$= 3^2 \times 3^6 \times \frac{1}{3^5}$

$= 3^{2+6-5}$

$(\because a^m \times a^n = a^{m+n}$ and $a^m \div a^n = a^{m-n})$

$= 3^{8-5}$

$= 3^3$

(iii) $p^3 \times p^{-10}$

$= p^3 \times \frac{1}{p^{10}}$

$= \frac{p^3}{p^{10}}$

$= p^{3-10}$ $\quad (\because a^m \div a^n = a^{m-n})$

$= p^{-7}$

(iv) $2^4 \times (-4)^{-2}$ LearnCBSE.in

$= 2^4 \times \frac{1}{(-4)^2}$

$= 2^4 \times \frac{1}{16}$

$= 2^4 \times \frac{1}{2^4}$

$= 2^{4-4}$ $\quad (\because a^m \div a^n = a^{m-n})$

$= 2^0$

$= 1$

(v) $8^p \times 8^q$

$= 8^{p+q}$ $\quad (\because a^m \times a^n = a^{m+n})$

Intext Questions

Question 1.
How many times larger than 4^{-2} is 4^2?
Solution:

$4^2 = 16$

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$\frac{4^2}{4^{-2}} = 4^2 \times 4^2$

$= 4^{2+2}$

$= 4^4$

$= 256$

4^2 is 256 (4^4) times larger than 4^{-2}.

Question 2.
Use the power line for 7 to answer the following questions.

7^7	823543
7^6	117649
7^5	16807
7^4	2401
7^3	343
7^2	49
7^1	7
7^0	1
7^{-1}	$\frac{1}{7}$
7^{-2}	$\frac{1}{49}$
7^{-3}	$\frac{1}{343}$
7^{-4}	$\frac{1}{2401}$

$2{,}401 \times 49 =$

$49^3 =$

$343 \times 2{,}401 =$

$\frac{16{,}807}{49} =$

$\frac{7}{343} =$

$\frac{16{,}807}{8{,}23{,}543} =$

$1{,}17{,}649 \times \frac{1}{343} =$

$\frac{1}{343} \times \frac{1}{343} =$

Solution:

(i) 2401×49

$= 7^4 \times 7^2$

$= 7^{4+2}$

$= 7^6$

$= 117649$

(ii) $49^3 = (7^2)^3 = 7^6 = 117649$

(iii) $343 \times 2401 = 7^3 \times 7^4 = 7^{3+4}$

$= 7^7 = 823543$

(iv) $\frac{16807}{49} = \frac{7^5}{7^2} = 7^{5-2} = 7^3 = 343$

(v) $\frac{7}{343} = \frac{7}{7^3} = 7^{1-3} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

(vi) $\frac{16807}{823543} = \frac{7^5}{7^7} = 7^{5-7} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

(vii) $1{,}17{,}649 \times \frac{1}{343} = 7^6 \times 7^{-3}$

$= 7^{6-3}$

$= 7^3$

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$= 343$

(viii) $\frac{1}{343} \times \frac{1}{343}$

$= \frac{1}{7^3} \times \frac{1}{7^3}$

$= \frac{1}{7^{3+3}}$

$= \frac{1}{7^6} = \frac{1}{117649}$

Question 3.
Write these numbers in the power of 10:
(i) 172
(ii) 5642
(iii) 6374
Solution:
(i) 172

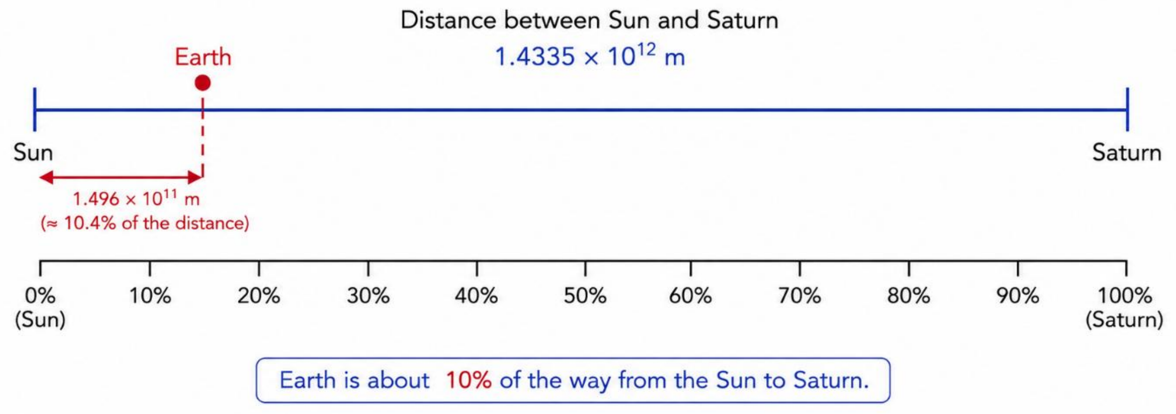
$= 1 \times 100 + 7 \times 10 + 2 \times 1$
$= (1 \times 10^2) + (7 \times 10^1) + (2 \times 10^0)$

(ii) 5642
$= 5 \times 1000 + 6 \times 100 + 4 \times 10 + 2 \times 1$
$= (5 \times 10^3) + (6 \times 10^2) + (4 \times 10^1) + (2 \times 10^0)$

(iii) 6374
$= 6 \times 1000 + 3 \times 100 + 7 \times 10 + 4 \times 1$
$= (6 \times 10^3) + (3 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$

Intext Questions9

1. The number line below shows the distance between the Sun and Saturn (1.4335×10^{12} m). On the number line below, mark the relative position of the Earth. The distance between the Sun and the Earth is 1.496×10^{11} m.



Question 2.
Express the following numbers in standard form.
(i) 59,853
(ii) 65,950
(iii) 34,30,000
(iv) 70,04,00,00,000
Solution:
(i) $59853 = \times 10000 = 5.9853 \times 10^4$
(ii) $65950 = \times 10000 = 6.595 \times 10^4$
(iii) $3430000 = \times 1000000 = 3.43 \times 10^6$
(iv) $70040000000 = \times 10000000000 = 7.004 \times 10^{10}$

Intext Questions

Question 2.
Calculate and write the answer using scientific notation. (Pages 38-39)
(i) How many ants are there for every human in the world?
(ii) If a flock of starlings contains 10,000 birds, how many flocks could there be in the world?
(iii) If each tree had about 104 leaves, find the total number of leaves on all the trees in the world.
(iv) If you stacked sheets of paper on top of each other, how many would you need to reach the Moon?
Solution:

(i) Estimated number of ants = 2×10^{16} (20 quadrillion)

Estimated human population = 8×10^{9}

Number of ants per person = $\frac{2 \times 10^{16}}{8 \times 10^{9}}$

$= 0.25 \times 10^{7}$

$= 2.5 \times 10^{6}$

(ii) Estimated number of starlings = 3.1×10^{8} (310 millions)

Number of birds in each flock = 1×10^{4}

Number of flocks in the world = $\frac{3.1 \times 10^{8}}{1 \times 10^{4}} = 3.1 \times 10^{4}$

(iii) Estimated number of trees = 3×10^{12}

Number of leaves on each tree = 1×10^{4}

Total number of leaves on all the trees in the world = $3 \times 10^{12} \times 1 \times 10^{4} = 3 \times 10^{16}$

(iv) Distance of Moon = 3.84×10^{8} m

Thickness of one sheet of paper = 1×10^{-4} m

Number of sheets required to reach moon = $\frac{3.84 \times 10^{8}}{1 \times 10^{-4}} = 3.84 \times 10^{12}$

2. A Different Way to Say Your Age!

(a) Roxie is 4840 days old. How old is she?

Estimate:

1 year ≈ 365 days

$$4840 \div 365 \approx 13.26$$

So, Roxie is about **13 years old** (which matches the statement "I completed 13 years a few weeks ago").

(b) Estu is 4070 days old. Find her date of birth.

First estimate her age:

$$4070 \div 365 \approx 11.15$$

So Estu is about **11 years old**.

Using today's date (**5 June 2026**):

- 11 years = $11 \times 365 = 4015$days
- Difference = $4070 - 4015 = 55$days

Count back 55 days from 5 June 2026:

- 31 days back → 5 May 2026
- Remaining 24 days back → **11 April 2026**

Since we went back 11 years and 55 days:

Date of Birth = 11 April 2015

Answer: Estu was born on **11 April 2015**.

(c) Roxie says, "I'm ______ hours old!"

Given:

$$4840 \text{ days}$$

Since 1 day = 24 hours,

$$4840 \times 24 = 116160$$

Roxie is 116,160 hours old.

Intext Questions

Calculate and write the answer using scientific notation:
(i) If one star is counted every second, how long would it take to count all the stars in the universe? Answer in terms of the number of seconds using scientific notation.
(ii) If one could drink a glass of water (200 ml) every 10 seconds, how long would it take to finish the entire volume of water on Earth? (Page 42)
Solution:
(i) Estimated number of stars in the observable universe: 1×10^{24} stars (1 septillion)
If you count 1 star per second, then:
Time required = 1×10^{24} seconds

(ii) Total volume of water on Earth: 1.386×10^9 km^3
Convert to milliliters: 1.386×10^9 km^3
$= 1.386 \times 10^9 \times 10^{15}$ ml
$= 1.386 \times 10^{24}$ ml
Capacity of each glass = 200 ml
Time taken to drink per glass = 10 seconds
Total number of glasses = = 6.93×10^{21}
Total time used = $6.93 \times 10^{21} \times 10 = 6.93 \times 10^{22}$ seconds

Figure It Out

Question 1.
Find out the units digit in the value of $2^{224} \div 4^{32}$? [Hint: $4 = 2^2$]
Solution:
$2^{224} \div 4^{32}$
For powers of 2, the units digits cycle as 2, 4, 8, 6.
($\because 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16 \Rightarrow 6, 2^5 = 32 \Rightarrow 2$)
This cycle has a length of 4.
The exponent is 224.
Divide 224 by 4: = 56
The remainder is 0.
When the remainder is 0, the units digit is the last in the cycle.
The last digit in the cycle 2, 4, 8, 6 is 6.

Question 2.
There are 5 bottles in a container. Every day, a new container is brought in. How many bottles would be there after 40 days?
Solution:
Start with 1 container having 5 bottles,

1 new container is added every day, so after 40 days.
Number of bottles after 40 days = 40 containers × 5 bottles each = 200 bottles

Question 3.
Write the given number as the product of two or more powers in three different ways. The powers can be any integers.
(i) 64^3
(ii) 192^8
(iii) 32^{-5}
Solution:

Number	Way 1	Way 2	Way 3
(*i*) 64^3	2^{18}	8^6	4^9
(*ii*) 192^8	$2^{48} \times 3^8$	$64^8 \times 3^8$	$16^8 \times 12^8$
(*iii*) 32^{-5}	2^{-25}	$8^{-5} \times 4^{-5}$	$\frac{1}{2^{25}}$

Question 4.
Examine each statement below and find out if it is 'Always True', 'Only Sometimes True', or 'Never True'. Explain your reasoning.
(i) Cube numbers are also square numbers.
(ii) Fourth powers are also square numbers.
(iii) The fifth power of a number is divisible by the cube of that number.
(iv) The product of two cube numbers is a cube number.
(v) q^{46} is both a 4th power and a 6th power (q is a prime number).
Solution:
(i) Cube numbers are also square numbers — Only Sometimes True
Reason:
(a) $64 = 4^3 = 8^2$ → both cube and square
(b) $8 = 2^3$ → not a square.
A number must be both a square and a cube, i.e., a sixth power, to satisfy both. Not all cubes are sixth powers.

(ii) Fourth powers are also square numbers. — Always True
Reason: Any fourth power is the form of a4 = (a2)2, which is clearly a square. Fourth powers are squares because squaring a square gives a fourth power.

(iiii) The fifth power of a number is divisible by the cube of that number. — Always True
Reason: a5 ÷ a2 = a5-3 = a2, which is valid for any a ≠ 0.
The fifth power contains at least three powers of the base, so it's divisible by its cube.

(iv) The product of two cube numbers is a cube number. — Always True
Reason: The product of two cubes is also a cube, just raise the product of their bases to the third power.

(v) q^{46} is both a 4th power and a 6th power (q is a prime number) — Never True
Reason: 46 is not divisible by 4 or 6 → can't be a 4th or 6th power.

Question 5.
Simplify and write these in the exponential form.
(i) $10^{-2} \times 10^{-5}$
(ii) $5^7 \div 5^4$
(iii) $9^{-7} \div 9^4$
(iv) $(13^{-2})^{-3}$
(v) $m^5 n^{12} (mn)^9$
Solution:
(i) $10^{-2} \times 10^{-5} = 10^{-2-5} = 10^{-7}$
$(a^m \times a^n = a^{m+n})$
(ii) $5^7 \div 5^4 = 5^{7-4} = 5^3$
$(a^m \div a^n = a^{m-n})$
(iii) $9^{-7} \div 9^4 = 9^{-7-4} = 9^{-11}$
$(a^m \div a^n = a^{m-n})$
(iv) $(13^{-2})^{-3} = (13)^{-2\times(-3)} = (13)^6$
$[(a^m)^n = a^{mn}]$
(v) $m^5n^{12} (mn)^9 = m^5n^{12}\, m^9n^9 = m^{5+9} \cdot n^{12+9}$
$= m^{14}n^{21}$ $[(a^m)^n = a^{mn}$ & $a^m \times a^n = a^{m+n}]$

Question 6.
If $12^2 = 144$, what is
(i) $(1.2)^2$
(ii) $(0.12)^2$
(iii) $(0.012)^2$
(iv) 120^2
Solution:
(i) $(1.2)^2 = = 1.44$
(11) $(0.12)^2 = = 0.0144$
(111) $(0.012)^2 = = 0.000144$
(iv) $120^2 = (12 \times 10)^2 = 144 \times 100 = 14400$

Question 7.
Circle the numbers that are the same
$2^4 \times 3^6$
$6^4 \times 3^2$
6^{10}
$18^2 \times 6^2$
6^{24}
Solution:

$6^4 \times 3^2 = (2 \times 3)^4 \times 3^2 = 2^4 \times 3^4 \times 3^2 = 2^4 \times 3^6$

$6^{10} = (2 \times 3)^{10} = 2^{10} \times 3^{10}$
$18^2 \times 6^2 = (3^2 \times 2)^2 \times (2 \times 3)^2$

$= 3^4 \times 2^2 \times 2^2 \times 3^2 = 2^4 \times 3^6$

$6^{24} = (2 \times 3)^{24} = 2^{24} \times 3^{24}$

Question 8.
Identify the greater number in each of the following:
(i) 4^3 or 3^4
(ii) 2^8 or 8^2
(iii) 100^2 or 2^{100}
Solution:
(i) $4^3 = 64$, $3^4 = 81$
$81 > 64$
$\therefore 3^4 > 4^3$

(ii) $2^8 = 256$, $8^2 = 64$
$256 > 64$
$\therefore 2^8 > 8^2$

(iii) $100^2 = 10{,}000$
$2^{100} \sim 1.27 \times 10^{30}$
A number with 30 zeros is much larger than 10,000.
$\therefore 2^{100} > 100^2$

Question 9.
A dairy plans to produce 8.5 billion packets of milk in a year. They want a unique ID (identifier) code for each packet. If they choose to use the digits 0-9, how many digits should the code consist of?
Solution:
Total no. of packets produced in a year = 8.5 billion

∴ Number of codes required = 8.5×10^9 codes
For ID digits to be taken from 0 to 9.
So each digit has 10 choices.
To make code with n digits, the total number of possible codes = 10^n
∴ $10^n \geq 8.5 \times 10^9$
10^9 = 1,00,00,00,000
The smallest value of n that satisfies
$10^n \geq 8.5 \times 10^9$ is 10.
Hence number of digits the code should consist of 10^{10}.

Question 10.
64 is a square number (8^2) and a cube number (4^3). Are there other numbers that are both squares and cubes? Is there a way to describe such numbers in general?
Solution:

Number	Form	Square of	Cube of
1	1^6	1^2	1^3
64	2^6	8^2	4^3
729	3^6	27^2	9^3
4096	4^6	64^2	16^3
15625	5^6	125^2	25^3

Yes, other numbers are both squares and cubes.
General Rule: A number is both a perfect square and a perfect cube if and only if it is a perfect sixth power, i.e., it can be written as x^6 for some integer x.

Question 11.
A digital locker has an alphanumeric (it can have both digits and letters) pass code of length 5. Some example codes are G89P0, 38098, BRJKW, and 003AZ. How many such codes are possible?
Solution:
Number of letters (A-Z) = 26
Number of digits (0-9) = 10
So, each character in the passcode can be any of the 36 alphanumeric characters.
Also, each of the 5 positions has 36 options.
Total codes = 36^5 = 6,04,66,176
Hence number of possible 5-character alphanumeric passcodes is 6,04,66,176.

Question 12.
The worldwide population of sheep (2024) is about 10^9, and that of goats is also about the same. What is the total population of sheep and goats?
(i) 20^9
(ii) 10^{11}
(iii) 10^{10}

(iv) 10^{18}
(v) 2×10^9
(vi) $10^9 + 10^9$
Solution:
Sheep population = 10^9
Goat population = 10^9
Total population of sheep and goats = $10^9 + 10^9 = 2 \times 10^9$

Question 13.
Calculate and write the answer in scientific notation:
(i) If each person in the world had 30 pieces of clothing, find the total number of pieces of clothing.
(ii) There are about 100 million bee colonies in the world. Find the number of honeybees if each colony has about 50,000 bees.
(iii) The human body has about 38 trillion bacterial cells. Find the bacterial population residing in all humans in the world.
(iv) Total time spent eating in a lifetime in seconds.
Solution:
(i) World population = 8.2 billion = 8.2×10^9
Pieces of clothing = 30 pieces per person
Total pieces of clothing = $8.2 \times 10^9 \times 30$
$= 246 \times 10^9$
$= 2.46 \times 10^{11}$ pieces of clothing

(ii) No. of bee colonies = 100 million = 1×10^8
No. of bee per colony = 50,000 = 5×10^4
Total no. of bees = $1 \times 10^8 \times 5 \times 10^4$
$= 5 \times 10^{8+4}$
$= 5 \times 10^{12}$ honeybees

(iii) No. of bacterial cells per human body = 38 trillion = 3.8×10^{13}
World population (approx.) = 8.2 billion = 8.2×10^9
Total bacterial population = $3.8 \times 10^{13} \times 8.2 \times 10^9$
$= 31.16 \times 10^{13+9}$
$= 31.16 \times 10^{22}$

(iv) Let average eating time per day = 1.5 hours
In seconds = $1.5 \times 60 \times 60 = 5400 = 5.4 \times 10^3$
and an average person's lifetime (approx.) = 70 years
In seconds = $70 \times 365 \times 24 \times 60 \times 60$

$= 161,148,960,000$
$= 1.61 \times 10^{11}$
Total time spent in eating $= 5.4 \times 10^3 \times 1.61 \times 10^{11}$
$= 8.694 \times 10^{3+11}$
$= 8.7 \times 10^{14}$ seconds

Practice Time 2.1

Q1(i) Express $7 \times 7 \times 7 \times 7 \times 7$in exponential form

Solution By Steps

Step 1: Count the repeated factor

The number 7 is multiplied 5 times.

Step 2: Write in exponent form

$$7 \times 7 \times 7 \times 7 \times 7 = 7^5$$

Final Answer

$$\boxed{7^5}$$

Q1(ii)

$$a \times a \times b \times b \times b$$

Step 1: Count a's = 2

Step 2: Count b's = 3

$$\boxed{a^2b^3}$$

Q1(iii)

$$3 \times 3 \times 7 \times 7 \times 7 \times 11 \times 11$$
$$= 3^2 \times 7^3 \times 11^2$$

Answer

$$\boxed{3^2 \times 7^3 \times 11^2}$$

Q1(iv)

$$a \times a \times b \times b \times b \times c \times c \times c \times c$$
$$= \boxed{a^2 b^3 c^4}$$

Q2(i)

$$(-5)^3 \div (-5)^4$$

Step 1: Apply quotient rule

$$(-5)^{3-4}$$
$$= (-5)^{-1}$$

Step 2: Convert negative exponent

$$= \frac{1}{-5}$$

Answer

$$\boxed{-\frac{1}{5}}$$

Q2(ii)

$$(-2)^4 \times \frac{1}{2^5}$$
$$= \frac{2^4}{2^5}$$

$$= 2^{-1}$$
$$= \frac{1}{2}$$

Answer

$$\boxed{\frac{1}{2}}$$

Q2(iii)

$$3^{-2} \times (5^{-3} \div 5^{-2})$$
$$= 3^{-2} \times 5^{-1}$$
$$= \frac{1}{3^2 \times 5}$$
$$= \frac{1}{45}$$

Answer

$$\boxed{\frac{1}{45}}$$

Q2(iv)

$$\left(3^{-2}\right)^{-2} \div \frac{1}{3^{-4}}$$
$$= 3^4 \div 3^4$$
$$= 1$$

Answer

$$\boxed{1}$$

Q2(v)

$$(3^{-1} \times 4^{-1}) \div 3^{-2}$$
$$= 3^{-1-(-2)} \times 4^{-1}$$
$$= 3^{1} \times 4^{-1}$$
$$= \frac{3}{4}$$

Answer

$$\boxed{\frac{3}{4}}$$

Q3

Initial flowers = 2

They double every day.

On the 15th day:

$$2 \times 2^{14} = 2^{15}$$

Answer

$$\boxed{2^{15} = 32768}$$

Q4

$$2^{5} \times 5^{5} = (2 \times 5)^{5} = 10^{5}$$

Hence the statement is **True**.

Answer

$$\boxed{\text{True}}$$

Q5

$$2^2 \times 2^3 = 2^{2+3} = 2^5$$

But

$$5^2 \times 5^3 = 5^5$$

not 25^5.

Therefore

$$\boxed{\text{No, } 5^2 \times 5^3 = 5^5}$$

Q6

4 positions, each can be filled by 26 letters.

$$\begin{aligned} &26 \times 26 \times 26 \times 26 \\ &= 26^4 \end{aligned}$$

Answer

$$\boxed{26^4 = 456976}$$

Q7

$$\begin{aligned} &2400 = 24 \times 100 \\ &= 2^3 \times 3 \times (2^2 \times 5^2) \\ &= 2^5 \times 3 \times 5^2 \end{aligned}$$

Answer

$$\boxed{2^5 \times 3 \times 5^2}$$

Q8

Compare:

$$3^5 = 243$$
$$9^2 = 81$$

Since $243 > 81$,

$$\boxed{3^5 > 9^2}$$

Q9(i)

$$33540$$
$$= 3 \times 10^4 + 3 \times 10^3 + 5 \times 10^2 + 4 \times 10^1$$

Q9(ii)

$$126400$$
$$= 1 \times 10^5 + 2 \times 10^4 + 6 \times 10^3 + 4 \times 10^2$$

Practice Time 2.2

Q1. Write the following numbers in standard form

Recall:

$$N = a \times 10^n (1 \leq a < 10)$$

(i) 0.000568

Solution By Steps

Step 1: Move the decimal point

$$0.000568 = 5.68$$

Decimal moved 4 places right.

Step 2: Write in standard form

$$0.000568 = 5.68 \times 10^{-4}$$

Final Answer

$$\boxed{5.68 \times 10^{-4}}$$

(ii) 3200000

Move decimal 6 places left:

$$3200000 = 3.2 \times 10^{6}$$

Answer

$$\boxed{3.2 \times 10^{6}}$$

(iii) 26700000

Move decimal 7 places left:

$$26700000 = 2.67 \times 10^{7}$$

Answer

$$\boxed{2.67 \times 10^{7}}$$

(iv) 0.000125

Move decimal 4 places right:

$$0.000125 = 1.25 \times 10^{-4}$$

Answer

$$\boxed{1.25 \times 10^{-4}}$$

(v) 2300000

$$2300000 = 2.3 \times 10^6$$

Answer

$$\boxed{2.3 \times 10^6}$$

(vi) 0.000915

$$0.000915 = 9.15 \times 10^{-4}$$

Answer

$$\boxed{9.15 \times 10^{-4}}$$

Q2. Find the standard form of

$$0.0005 + 0.0085$$

Solution By Steps

Step 1: Add the numbers

$$0.0005 + 0.0085 = 0.009$$

Step 2: Convert to standard form

$$0.009 = 9 \times 10^{-3}$$

Final Answer

$$\boxed{9 \times 10^{-3}}$$

Q3. Express the product in standard form

$$1.3 \times 10^4 \times 3.4 \times 10^{-2}$$

Solution By Steps

Step 1: Multiply coefficients

$$1.3 \times 3.4 = 4.42$$

Step 2: Multiply powers of 10

$$10^4 \times 10^{-2} = 10^{4-2} = 10^2$$

Step 3: Combine

$$4.42 \times 10^2$$

Final Answer

$$\boxed{4.42 \times 10^2}$$

Q4. Express in standard form

(i) Speed of light

$$299792458 \text{ m/s}$$

Move decimal 8 places left.

$$2.99792458 \times 10^8$$

Answer

$$\boxed{2.99792458 \times 10^8 \text{ m/s}}$$

(ii) Thickness of a human hair

$$0.000076 \text{ m}$$

Move decimal 5 places right.

$$7.6 \times 10^{-5}$$

Answer

$$\boxed{7.6 \times 10^{-5} \text{ m}}$$

(iii) One Angstrom

$$0.0000000001 \text{ m}$$

Move decimal 10 places right.

$$1 \times 10^{-10}$$

Answer

$$\boxed{1 \times 10^{-10} \text{ m}}$$

(iv) One nanometre

$$\frac{1}{1000000000}$$
$$= \frac{1}{10^9}$$
$$= 10^{-9}$$

Answer

$$\boxed{1 \times 10^{-9} \text{ m}}$$

Q5. Estimate the number of steps to reach Mars

Distance to Mars:

$$2.25 \times 10^8 \text{ km}$$

Convert into metres:

$$2.25 \times 10^8 \times 10^3$$
$$= 2.25 \times 10^{11} \text{ m}$$

One step:

$$0.5 \text{ m}$$

Number of steps:

$$\frac{2.25 \times 10^{11}}{0.5}$$
$$= 4.5 \times 10^{11}$$

Final Answer

$$\boxed{4.5 \times 10^{11} \text{ steps}}$$

EXAM TIME – Multiple Choice Questions (MCQs)

Q1. The value of 2^4

Solution

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Answer: (c) 16

Q2. The value of 10^{-10}

Using

$$10^{-n} = \frac{1}{10^n}$$
$$10^{-10} = \frac{1}{10^{10}}$$

Answer: (a)

Q3. The value of $5^2 \times 5^{-2}$

$$5^2 \times 5^{-2} = 5^{2+(-2)}$$
$$= 5^0 = 1$$

Answer: (a) 1

Q4.

$$(-9)^3 \div (-9)^8$$
$$= (-9)^{3-8}$$
$$= (-9)^{-5}$$

Answer: (b)

Q5. Exponential form of 625

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

Answer: (c)

Q6. Lock with 4 slots and digits 0–9

Each slot has 10 choices.

$$10 \times 10 \times 10 \times 10 = 10^4$$

Answer: (b)

Q7. Trees and leaves

$$(3 \times 10^{12})(10^4)$$
$$= 3 \times 10^{16}$$

Answer: (a)

Q8. Standard form of 0.000072

$$0.000072 = 7.2 \times 10^{-5}$$

Answer: (d)

Q9.

$$2.08 \times 10^{-5}$$

Move decimal 5 places left.

$$= 0.0000208$$

Answer: (a)

Q10.

$$51.26 = 5.126 \times 10^{1}$$

Answer: (b)

B. Fill in the Blanks

1.

Initial thickness = 0.001 cm

After 7 folds:

$$\begin{aligned} &0.001 \times 2^{7} \\ &= 0.001 \times 128 \\ &= 0.128 \end{aligned}$$

Answer: $\boxed{0.128 \text{ cm}}$

2.

$$5^0 = 1$$

Answer: $\boxed{1}$

3.

Prime factorization of 450

$$\begin{aligned} 450 &= 45 \times 10 \\ &= (3^2 \times 5)(2 \times 5) \\ &= 2 \times 3^2 \times 5^2 \end{aligned}$$

Answer:

$$\boxed{2 \times 3^2 \times 5^2}$$

4.

One crore

$$\begin{aligned} &= 10000000 \\ &= 10^7 \end{aligned}$$

Answer: $\boxed{10^7}$

5.

$$\begin{aligned} &12345000 \\ &= 1.2345 \times 10^7 \end{aligned}$$

Answer: $\boxed{1.2345 \times 10^7}$

C. True / False

1.

$$n^4 = (n^2)^2$$

Always a perfect square.

Answer: True

2.

$$3^{-2} = \frac{1}{3^2}$$

Answer: True

3.

When dividing powers of same base, exponents are subtracted.

Statement says multiplied.

Answer: False

4.

$$59000 = 5.9 \times 10^4$$

Answer: True

5.

$$648 = 2^3 \times 3^4$$

not $2^4 \times 3^4$

Answer: False

D. Match the Columns

(a)

$$2^4 \div 2^6$$
$$= 2^{-2}$$
$$= \frac{1}{4}$$

→ (ii)

(b)

$$a^3 \times a^5$$
$$= a^8$$

→ (iii)

(c)

$$3^{-2}$$
$$= \frac{1}{9}$$

→ (iv)

(d)

$$4^3 \times 4^{-3}$$
$$= 4^0$$
$$= 1$$

→ (i)

Matching:

$$\boxed{(a) \rightarrow (ii),\ (b) \rightarrow (iii),\ (c) \rightarrow (iv),\ (d) \rightarrow (i)}$$

E. Very Short Answer Questions

1.

Thickness after n folds:

Each fold doubles thickness.

$$\boxed{t \times 2^n}$$

2.

$$[2^{-1} \times 3^{-1}]^{-1}$$
$$= \left(\frac{1}{6}\right)^{-1}$$
$$= 6$$

Answer: $\boxed{6}$

3.

$$13^{-13} \div 13^{13}$$
$$= 13^{-26}$$
$$= \frac{1}{13^{26}}$$

Answer

$$\boxed{\frac{1}{13^{26}}}$$

4.

$$3250000000$$
$$= 3.25 \times 10^9$$

Answer: $\boxed{3.25 \times 10^9}$

5.

$$\begin{aligned} &0.0000056789 \\ &= 5.6789 \times 10^{-6} \end{aligned}$$

Answer: $\boxed{5.6789 \times 10^{-6}}$

6.

$$\begin{aligned} &2.8 \times 10^{-10} \\ &= 0.00000000028 \end{aligned}$$

Answer: $\boxed{0.00000000028}$

7.

Expand 4050

$$4050 = 4 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 0 \times 10^0$$

Answer

$$\boxed{4 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 0 \times 10^0}$$

F. Short Answer Questions

1. Bacteria doubles every hour

Initial = 1 cell

After 12 hours:

$$\begin{aligned} &1 \times 2^{12} \\ &= \boxed{2^{12}} \end{aligned}$$

2(i)

$$\left[\left(-\frac{3}{2}\right)^{-2}\right]^{-3}$$

Power of power:

$$=\left(-\frac{3}{2}\right)^{6}$$

Express with negative exponent:

$$=\boxed{\left(-\frac{2}{3}\right)^{-6}}$$

2(ii)

$$(2^5 \div 2^8) \times 2^{-7}$$
$$= 2^{-3} \times 2^{-7}$$
$$= 2^{-10}$$

Answer

$$\boxed{2^{-10}}$$

3. Seconds in 3.8 months

$$3.8 \times 2.6 \times 10^6$$
$$= 9.88 \times 10^6$$

Answer

$$\boxed{9.88 \times 10^6}$$

4.

$$\frac{1.5 \times 10^6}{2.5 \times 10^4}$$
$$= \frac{1.5}{2.5} \times 10^2$$
$$= 0.6 \times 10^2$$
$$= 6 \times 10^1$$

Answer

$$\boxed{6 \times 10^1}$$

---

5. Electron mass in grams

$$9.1093826 \times 10^{-31} \text{ kg}$$

Since

$$1 \text{ kg} = 10^3 \text{ g}$$
$$9.1093826 \times 10^{-31} \times 10^3$$
$$= 9.1093826 \times 10^{-28} \text{ g}$$

Answer

$$\boxed{9.1093826 \times 10^{-28} \text{ g}}$$

G. Long Answer

Question 1

Bacteria doubles every 30 minutes

There are 2 doublings per hour.

(a) After 10 h

Number of doublings:

$$10 \times 2 = 20$$

Cells:

$$2^{20}$$
$$= 1048576$$

Answer

$$\boxed{2^{20} = 1048576}$$

(b) After 25 h

Number of doublings:

$$25 \times 2 = 50$$

Cells:

$$\boxed{2^{50}}$$

(ii)

The growth pattern is:

$$1,2,4,8,16, \ldots$$

This is **exponential growth**.

Question 2

Given:

$$a = -1, b = 2$$

(i) $a^b + b^a$

Solution By Steps

$$a^b = (-1)^2 = 1$$
$$b^a = 2^{-1} = \frac{1}{2}$$

Adding:

$$1 + \frac{1}{2} = \frac{3}{2}$$

Final Answer

$$\boxed{\frac{3}{2}}$$

(ii) $a^b - b^a$

$$(\quad -1)^2 - 2^{-1}$$
$$= 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

Final Answer

$$\boxed{\frac{1}{2}}$$

(iii) $a^b \times b^a$

$$(-1)^2 \times 2^{-1}$$
$$= 1 \times \frac{1}{2}$$
$$= \frac{1}{2}$$

Final Answer

$$\boxed{\frac{1}{2}}$$

(iv) $a^b + b^a$

Same as part (i)

$$1 + \frac{1}{2}$$
$$= \frac{3}{2}$$

Final Answer

$$\boxed{\frac{3}{2}}$$

G. Question 3

Convert each mass into standard form.

Recall:

$$N = a \times 10^n \ (1 \leq a < 10)$$

Sun

$$1990000000000000000000000000000$$
$$= 1.99 \times 10^{30}$$

Mercury

$$330000000000000000000000$$
$$= 3.3 \times 10^{23}$$

Venus

$$4870000000000000000000000$$
$$= 4.87 \times 10^{24}$$

Earth

$$5970000000000000000000000$$
$$= 5.97 \times 10^{24}$$

Mars

$$642000000000000000000000$$
$$= 6.42 \times 10^{23}$$

Jupiter

$$1900000000000000000000000000$$
$$= 1.90 \times 10^{27}$$

Saturn

$$568000000000000000000000000$$
$$= 5.68 \times 10^{26}$$

Uranus

$$86800000000000000000000000$$
$$= 8.68 \times 10^{25}$$

Neptune

$$\begin{aligned}&102800000000000000000000000\\&= 1.028 \times 10^{26}\end{aligned}$$

Pluto

$$\begin{aligned}&12700000000000000000000\\&= 1.27 \times 10^{22}\end{aligned}$$

Moon

$$\begin{aligned}&73500000000000000000000\\&= 7.35 \times 10^{22}\end{aligned}$$

Arrange planets from least to greatest mass

Compare powers of 10.

$\boxed{\text{Pluto} < \text{Mercury} < \text{Mars} < \text{Venus} < \text{Earth} < \text{Uranus} < \text{Neptune} < \text{Saturn} < \text{Jupiter}}$

Planet having approximately same mass as Earth

Earth:

$$5.97 \times 10^{24}$$

Venus:

$$4.87 \times 10^{24}$$

Closest value.

Answer

$$\boxed{\text{Venus}}$$

Question 4 (Crop Production)

Given Table

Crop	**2008 Production**
Bajra	1.4×10^3
Jowar	1.7×10^6
Rice	3.7×10^3
Wheat	5.1×10^5

Changes in 2009:

- Bajra : −100
- Jowar : −440000
- Rice : −100
- Wheat : +190000

(i) Which crop had the greatest decrease?

Compare decreases:

$$100,\ 440000,\ 100$$

Largest decrease:

$$\boxed{\text{Jowar}}$$

(ii) Production in 2009

Bajra

$$\begin{aligned} &1.4 \times 10^3 = 1400 \\ &1400 - 100 = 1300 \\ &= 1.3 \times 10^3 \end{aligned}$$

Jowar

$$1.7 \times 10^6 = 1700000$$
$$1700000 - 440000$$
$$= 1260000$$
$$= 1.26 \times 10^6$$

Rice

$$3700 - 100$$
$$= 3600$$
$$= 3.6 \times 10^3$$

Wheat

$$510000 + 190000$$
$$= 700000$$
$$= 7 \times 10^5$$

(iii) Rice production in 2019

Rice decreases by 100 hectares each year.

2009 production:

$$3600$$

From 2009 to 2019 = 10 years

Total decrease:

$$10 \times 100 = 1000$$

Production in 2019:

$$3600 - 1000$$
$$= 2600$$

Standard form:

$$2600 = 2.6 \times 10^3$$

Answer

$$\boxed{2.6 \times 10^3}$$

Competency-Based Questions

A. Assertion–Reason

Q1

Assertion:

"When a number is raised to power zero, result is always zero."

This is false because

$$a^0 = 1$$

Reason:

"Any number raised to power zero equals one."

True.

Answer

$$\boxed{(d)}$$

(A is false but R is true)

Q2

Assertion:

$$0.00042 = 4.2 \times 10^{-4}$$

True.

Reason:

In scientific notation coefficient lies between 1 and 10 and exponent tells decimal shift.

True and explains assertion.

Answer

$$\boxed{(a)}$$

B. Case Study

Distance Venus–Earth:

$$40000000 \text{ km}$$

Distance Earth–Mars:

$$225000000 \text{ km}$$

(i) Exponential form

$$40000000$$
$$= 4 \times 10^7$$

Answer

$$\boxed{4 \times 10^7 \text{ km}}$$

(ii) How much farther is Mars?

$$225000000 - 40000000$$
$$= 185000000$$
$$= 1.85 \times 10^8$$

Answer

$$\boxed{1.85 \times 10^8 \text{ km}}$$

(iii) Ratio

$$40000000 : 225000000$$

Divide by 5000000

$$= 8 : 45$$

Answer

$$\boxed{8 : 45}$$

Chapter 3 : A Story of Numbers

NCERT CORNER

Figure It Out

Question 1.
Suppose you are using the number system that uses sticks to represent numbers, as in Method 1. Without using either the number names or the numerals of the Hindu number system, give a method for adding, subtracting, multiplying, and dividing two numbers or two collections of sticks.
Solution:
Suppose there are two groups of sheep.
The first group has 6 sheep and the second has 4 sheep.
We have counted using pebbles.
To add, put all pebbles in the same pouch.
To subtract, take out as many pebbles as in the pouch having fewer pebbles from the pouch having more pebbles.
Now, suppose we wish to know how many sheep will be twice the number in the first group.

For this, we count twice using pebbles and put all the pebbles in the same pouch.
Suppose we have 12 pebbles and want to divide them into three equal groups. We put them one by one in three bowls till we exhaust all. The number of pebbles in each bowl gives the quotient.

Question 2.
One way of extending the number system in Method 2 is by using strings with more than one letter — for example, we could use ‘aa’ for 27. How can you extend this system to represent all the numbers? There are many ways of doing it!
Solution:
a, b, c, …, z are 26 numbers
aa, bb, …, zz are 26 more numbers.

Question 3.
Try making your own number system.
Solution:
Let base be 3
Then $3^0 = 1 = A$, $3^1 = 3 = B$, $3^2 = 9 = C$….,

1	A
2	AA
3	B
4	BA
5	BAA
6	BB
7	BBA
8	BBAA
9	C
10	CA
11	CAA
12	CB

Q.4 Quickly count the number of objects in each box.

By observing the pictures:

Box	**Objects**	**Number**
(i)	Ducks	**2**

Box	Objects	Number
(ii)	Flowers	**5**
(iii)	Owls	**4**
(iv)	Small squares	**6**
(v)	Dog	**1**
(vi)	Grapes	**About 10–12** (need counting)
(vii)	Apples	**6**
(viii)	Sticks/Pencils	**About 9–10** (need counting)
(ix)	Mountains	**2**

Answer:

The numbers that can be seen immediately without counting are:

1, 2, 4, 5 and 6

For larger groups such as grapes and sticks, counting is needed.

Up to what group size could you immediately see the number of objects without counting?

Usually, most people can identify groups of up to **4 or 5 objects** instantly without counting. This ability is called **subitizing**.

Q.5

What could be the difficulties with using a number system that counts only in groups of a single particular size?

Suppose a number system uses only groups of **5**.

Then:

- Very large numbers become difficult to write.
- Too many symbols are needed.
- Reading and comparing numbers becomes difficult.

- Calculations become lengthy and confusing.

How would you represent 1345 in a system that counts only by 5s?

We repeatedly divide by 5.

Step 1

1345 ÷ 5 = 269

Step 2

269 ÷ 5 = 53 remainder 4

Step 3

53 ÷ 5 = 10 remainder 3

Step 4

10 ÷ 5 = 2 remainder 0

Step 5

2 ÷ 5 = 0 remainder 2

Now write remainders from bottom to top:

20340_5

Verification

$$\begin{aligned}
&2 \times 5^4 + 0 \times 5^3 + 3 \times 5^2 + 4 \times 5^1 + 0 \times 5^0 \\
&= 2 \times 625 + 0 + 3 \times 25 + 4 \times 5 + 0 \\
&= 1250 + 75 + 20 \\
&= 1345
\end{aligned}$$

Answer:

$$\boxed{1345_{10} = 20340_5}$$

Figure It Out

1. Represent the following numbers in Roman Numerals

(i) 1222

$$1222 = 1000 + 200 + 20 + 2$$

Roman Numerals:

1000 = M

200 = CC

20 = XX

2 = II

Therefore,

$$\boxed{1222 = MCCXXII}$$

(ii) 2999

$$2999 = 2000 + 900 + 90 + 9$$

2000 = MM

900 = CM

90 = XC

9 = IX

Therefore,

$$\boxed{2999 = MMCMXCIX}$$

(iii) 302

$$302 = 300 + 2$$

300 = CCC

2 = II

Therefore,

$$\boxed{302 = CCCII}$$

(iv) 715

$$715 = 700 + 10 + 5$$

700 = DCC

10 = X

5 = V

Therefore,

$$\boxed{715 = DCCXV}$$

2. Add LXXXVII + LXXVIII

Convert to Hindu-Arabic numbers

LXXXVII

= 50 + 30 + 7

= 87

LXXVIII

= 50 + 20 + 8

= 78

Add

$$87 + 78 = 165$$

Convert 165 back to Roman Numerals

100 = C

50 = L

10 = X

5 = V

Therefore,

$$165 = CLXV$$

Answer:

$$\boxed{LXXXVII + LXXVIII = CLXV}$$

3. Find the products

(i) V × L

V = 5

L = 50

$$5 \times 50 = 250$$

250 = CCL

Answer

$$\boxed{V \times L = CCL}$$

(ii) L × D

L = 50

D = 500

$$50 \times 500 = 25000$$

Roman numerals are usually written only up to a few thousand in the standard school system.

So,

$$\boxed{L \times D = 25000}$$

(Extended Roman notation would be required.)

(iii) V × D

V = 5

D = 500

$$5 \times 500 = 2500$$

2500

= 2000 + 500

= MM + D

Answer

$$\boxed{V \times D = MMD}$$

(iv) VII × IX

VII = 7

IX = 9

$$7 \times 9 = 63$$

63

= 50 + 10 + 3

= L + X + III

Answer

$$\boxed{VII \times IX = LXIII}$$

Figure It Out

Question 1.
A group of indigenous people in a Pacific island uses different sequences of number names to count different objects. Why do you think they do this?
Solution:
Counting in twos is more efficient in representing numbers than, for example, a tally system.

Question 2.
Consider the extension of the Gumulgal number system beyond 6 in the same way of counting by 2s. Come up with ways of performing the different arithmetic operations (+, -, ×, ÷) for numbers occurring in this system, without using Hindu numerals. Use this to evaluate the following:

Solution:

(*i*) uk - uk - uk - uk - ur
+ uk - uk - uk - ur
uk - uk - uk - uk - uk - uk - uk - uk [ur + ur = uk]

(*ii*) uk - uk - uk - uk - ur
− uk - uk - uk
uk - ur

(*iii*) (uk - uk - uk - uk - ur) × (uk - uk)
= (uk - uk - uk - uk - ur) × uk + (uk - uk - uk - uk - ur) × uk
= uk - uk - uk - uk - uk - uk - uk - uk - uk (9 times)
+ uk - uk - uk - uk - uk - uk - uk - uk - uk (9 times)

= (uk - uk - .... uk) (18 times)

(*iv*) (uk - uk - uk - uk - uk - uk - uk - uk) ÷ (uk - uk)

= [(uk - uk) - (uk - uk) - (uk - uk) - (uk - uk)] ÷ (uk - uk)

= 4 × [(uk - uk) ÷ (uk - uk)] = $\underbrace{\text{ur}}_{\text{4 times}}$

= $\underbrace{\text{ur - ur}}$ - $\underbrace{\text{ur - ur}}$
= uk - uk

Question 3.
Identify the features of the Hindu number system that make it efficient when compared to the Roman number system.
Solution:
Hindu numbers have ‘0’ and a place value system, which Roman numerals do not have.
The Hindu number system is a positional system, whereas the Roman system is not.

Question 4.
Using the ideas discussed in this section, try refining the number system you might have made earlier.
Solution:
Try it yourself.

Figure It Out

Question 1.
Represent the following numbers in the Egyptian system:
10458, 1023, 2660, 784, 1111, 70707

Solution:

10458 = 10000 + 400 + 50 + 8

1023 = 1000 + 20 + 3

2660 = 2000 + 600 + 60

784 = 700 + 80 + 4

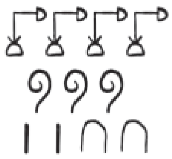
1111 = 1000 + 100 + 10 + 1

70707 = 70000 + 700 + 7

Question 2.
What numbers do these numerals stand for?

(i)

(ii)

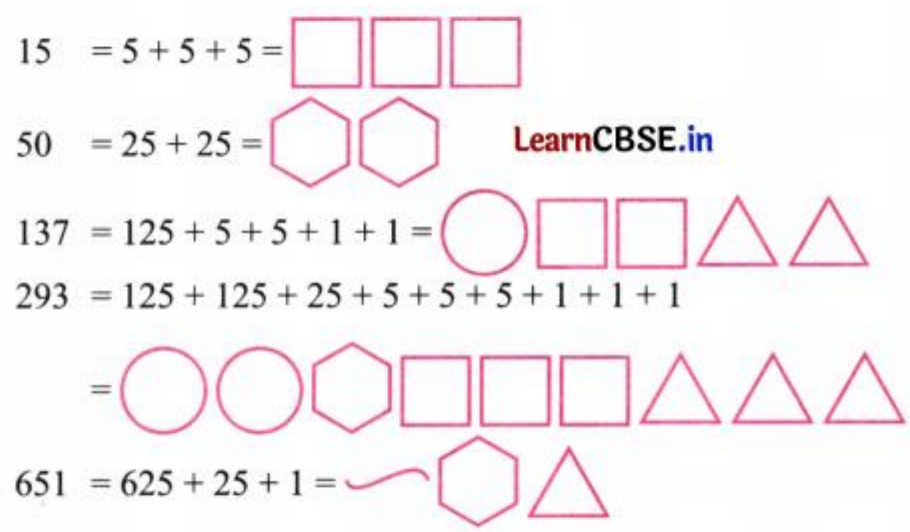


Solution:
(i) 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10 + 6 + 10
= 200 + 70 + 6
= 276

(ii) 1000 + 1000 + 1000 + 1000 + 100 + 100 + 100 + 1 + 1 + 10 + 10
= 4000 + 300 + 20 + 2
= 4322

Figure It Out

Question 1.
Write the following numbers in the above base-5 system using the symbols in Table 2: 15, 50, 137, 293, 651.
Solution:

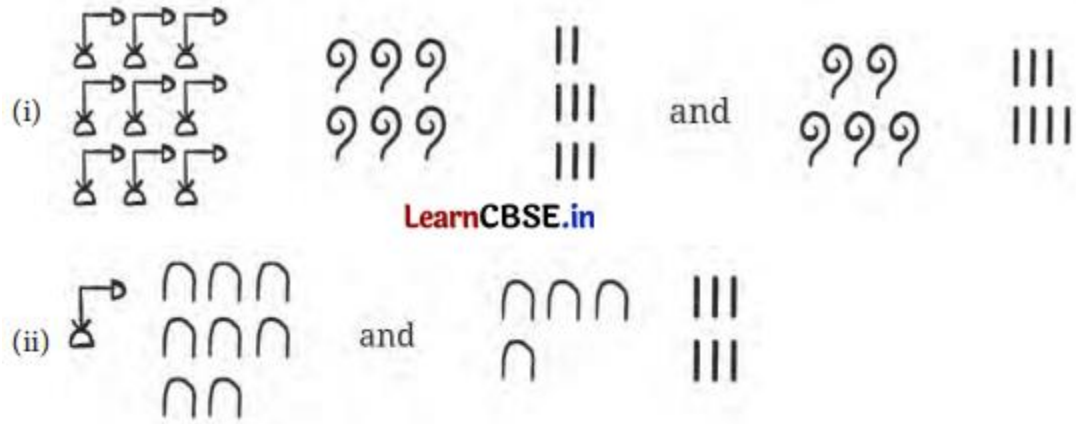


Question 2.
Is there a number that cannot be represented in our base-5 system above? Why or why not?
Solution:
Yes. Zero (0) cannot be represented in our base-5 system as there is no symbol for it.

Question 3.
Compute the landmark numbers of a base-7 system. In general, what are the landmark numbers of a base-n system?
Solution:
$7^0 = 1$, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$
Hence, 1, 7, 49, 343, 2401 are landmark numbers of base 7.
The landmark numbers of a base-n number system are the powers of n starting from $n^0 = 1, n, n^2, n^3, \ldots$

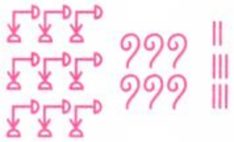
Figure It Out

Question 1.
Add the following Egyptian numerals:



Solution:

(i) Here,



$= 9 \times 1000 + 6 \times 100 + 8 \times 1$

$= 9000 + 600 + 8 = 9608$

and

$= 5 \times 100 + 7 \times 1 = 500 + 7 = 507$

$$\begin{array}{r} 9608 \\ +\ \ 507 \\ \hline 10115 \end{array}$$

10115 or 

(ii) Here

$= 1000 + 8 \times 10 = 1080$

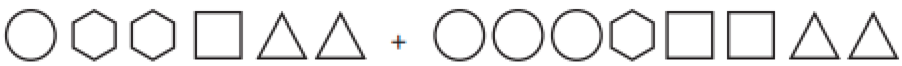
and

$= 4 \times 10 + 6 \times 1 = 40 + 6 = 46$

$$\begin{array}{r} 1080 \\ +\ \ 46 \\ \hline 1126 \end{array}$$

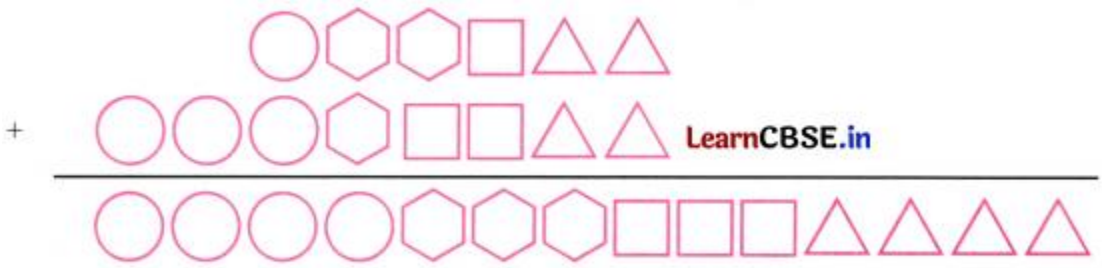
1126 or 

Question 2.
Add the following numerals that are in the base-5 system that we created:



Remember that in this system, 5 times a landmark number gives the next one!

Solution:



Q.3 What is any landmark number multiplied by ∩ (that is 10)?

Since ∩ = 10_5 = 5,

Multiplying a landmark number by ∩ shifts it to the next landmark.

(i) ∩ × ∩

$$10_5 \times 10_5 = 100_5$$

Answer: Spiral symbol (100_5)

(ii) Spiral × ∩

$$100_5 \times 10_5 = 1000_5$$

Answer: Lotus/flag symbol

(iii) Lotus × ∩

$$1000_5 \times 10_5 = 10000_5$$

Answer: Finger symbol

(iv) Finger × ∩

$$10000_5 \times 10_5 = 100000_5$$

Answer: Next landmark after finger symbol.

Q.4 What is any landmark number multiplied by Spiral (10^2)?

Since Spiral = $100_5 = 25$,

(i) ∩ × Spiral

$$10_5 \times 100_5 = 1000_5$$

Answer: Lotus symbol

(ii) Spiral × Spiral

$$100_5 \times 100_5 = 10000_5$$

Answer: Finger symbol

(iii) Lotus × Spiral

$$1000_5 \times 100_5 = 100000_5$$

Answer: Next landmark after finger symbol.

(iv) Finger × Spiral

$$10000_5 \times 100_5 = 1000000_5$$

Answer: Two landmarks after finger symbol.

Q.5 Find the following products

(i) ∩ × Spiral

Already done above:

$$10_5 \times 100_5 = 1000_5$$

Answer: Lotus symbol

(ii) Spiral × Lotus

$$100_5 \times 1000_5 = 100000_5$$

Answer: Next landmark after finger symbol

(iii) Lotus × Lotus

$$1000_5 \times 1000_5 = 1000000_5$$

Answer: Two landmarks after finger symbol

(iv) Finger × (next landmark)

$$10000_5 \times 100000_5 = 1000000000_5$$

Answer: A much higher landmark number (1 followed by 9 zeros in base 5).

Q.6

Does this property hold true?

Yes.

When we multiply powers of the base:

$$10^a \times 10^b = 10^{a+b}$$

This property works in the base-5 system and in **every positional number system**.

Answer

Yes, the property holds in the base-5 system and in any number system having a base.

Q.7 (i)

The numeral shown is:

- 5 spirals = 5×25
- 2 arches = 2×5
- 2 strokes = 2×1

So,

$$
\begin{aligned}
&5 \times 25 + 2 \times 5 + 2 \\
&= 125 + 10 + 2 \\
&= 137
\end{aligned}
$$

Convert 137 to base 5:

$$137 = 1022_5$$

Now multiply by ∩ (10_5):

$$1022_5 \times 10_5 = 10220_5$$

Answer

$$\boxed{10220_5}$$

(Symbolically, all digits shift one place left.)

Q.7 (ii)

Lotus × ∩ × ∩

$$
\begin{aligned}
&1000_5 \times 10_5 \times 10_5 \\
&= 100000_5
\end{aligned}
$$

Answer

$$\boxed{100000_5}$$

which is the **next landmark after the finger symbol**.

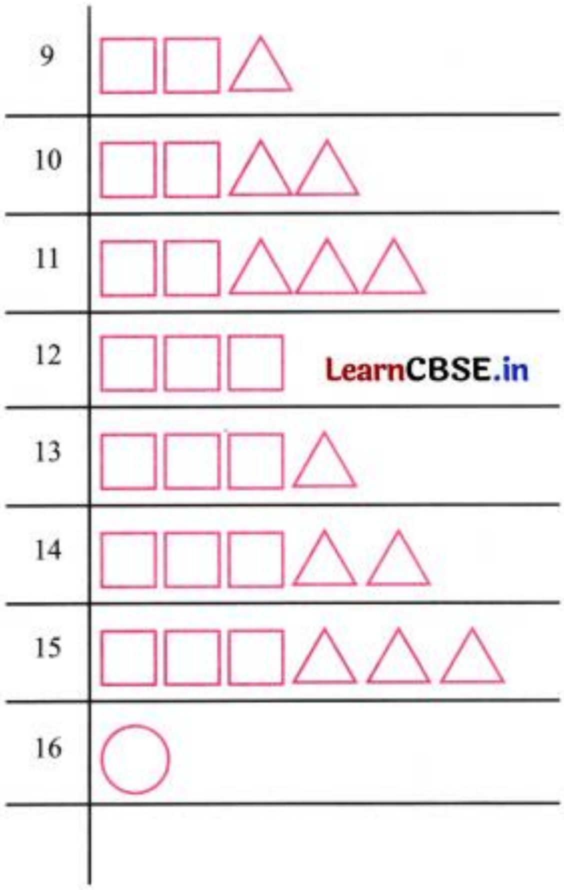
Figure It Out

Question 1.
Can there be a number whose representation in Egyptian numerals has one of the symbols occurring 10 or more times? Why not?
Solution:
No, ten times any landmark number will give the next landmark number.

Question 2.
Create your own number system of base 4, and represent numbers from 1 to 16.
Solution:

Let $4^0 = 1 =$ △, $4^1 = 4 =$ □, $4^2 = 16 =$ ○

1	△
2	△△
3	△△△
4	□
5	□△
6	□△△
7	□△△△
8	□□



Question 3.
Give a simple rule to multiply a given number by 5 in the base-5 system that we created.
Solution:
Rule: The Product of a landmark number with another landmark number gives a landmark number.

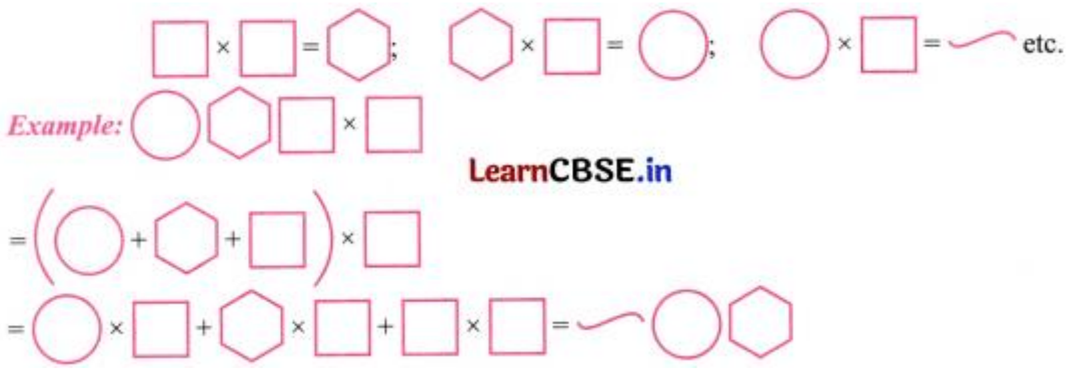


Figure It Out

Question 1.
Represent the following numbers in the Mesopotamian system:
(i) 63
(ii) 132

(iii) 200
(iv) 60
(v) 3605
Solution:

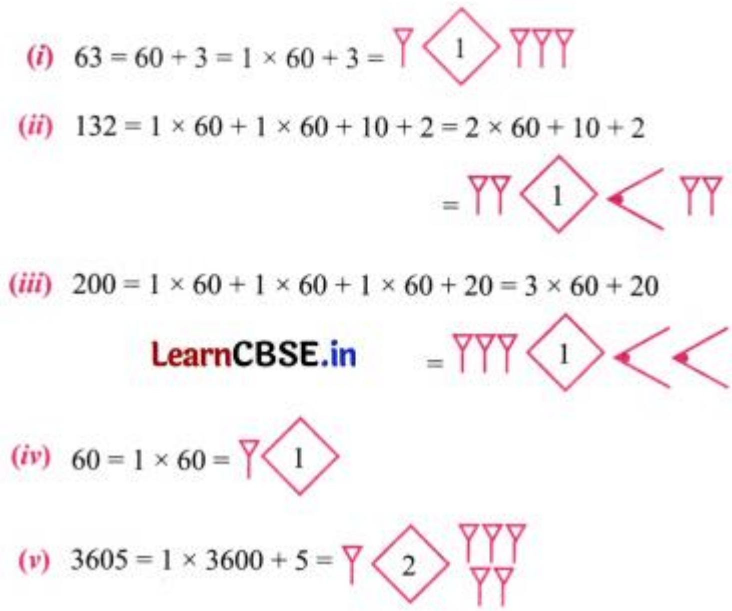


Figure It Out

Question 1.
Why do you think the Chinese alternated between the Zong and Heng symbols? If only the Zong symbols were to be used, how would 41 be represented? Could this numeral be interpreted in any other way if there is no significant space between two successive positions?
Solution:
The most appropriate reason seems to be that they wanted to avoid misinterpretation while reading the number, as all numerals are represented by rods.
Using only Zong symbols, 41 will be written as | | | | |
This can easily be misinterpreted as 5 if no significant space is left between successive positions.

Question 2.
Form a base-2 place value system using ‘ukasar’ and ‘urapon’ as the digits. Compare this system with that of the Gumulgal’s.
Solution:
Let $2^0 = 1 = A$, $2^1 = 2 = B$, $2^2 = 4 = C$, $2^4 = 16 = D$,….

Base 2-system	Gumulgal system
1 : A	ur
2 : B	uk
3 : BA	uk – ur
4 : C	uk – uk
5 : CA	uk – uk – ur
6 : CB	uk – uk – uk
7 : CBA	uk – uk – uk – ur
8 : D	uk – uk – uk – uk

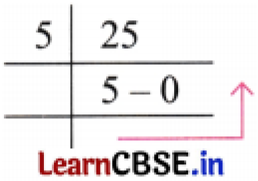
Both have the same base 2, but the base 2 system has many landmark numbers, whereas the Gumulgal system has only two landmark numbers.

Question 3.
Where in your daily lives, and in which professions, do the Hindu numerals, and 0, play an important role? How might our lives have been different if our number system and 0 hadn't been invented or conceived of?
Solution:
They are useful wherever we have to read unit numbers or do any calculation.
In case '0' was not there, all the above would have become much tedious and cumbersome.
Also, there would have been no computers.

Question 4.
The ancient Indians likely used base 10 for the Hindu number system because humans have 10 fingers, and so we can use our fingers to count. But what if we had only 8 fingers? How would we be writing numbers then? What would the Hindu numerals look like if we were using base 8 instead? Base 5? Try writing the base-10 Hindu numeral 25 as base-8 and base-5 Hindu numerals, respectively. Can you write it in base-2?
Solution:
For base 8, the numerals would have been: 0, 1, 2, 3, 4, 5, 6, 7
For base 5, the numerals would have been: 0, 1, 2, 3, 4

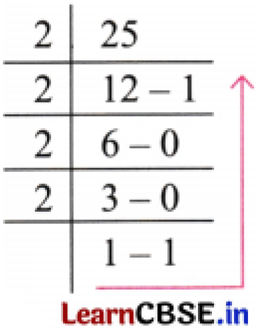
8	25
	3 – 1

$25_{10} = 31_8$
25 can be written as 31 in base 8.



$25_{10} = 50_5$

25 can be written as 50 in base 5.



$25_{10} = 11001_2$

25 can be written as 1101 in base 2.

EXAM TIME

A. Multiple Choice Questions

1. Which of the following was one of the earliest ways humans kept count?

(a) Writing on paper

(b) Counting on abacuses

(c) Using tally marks or notches on bones

(d) Using Roman numerals

Solution

Early humans did not have paper, abacuses, or numeral systems. They used marks on bones, sticks, and stones to count.

Answer

(c) Using tally marks or notches on bones

---

2. The symbol '0' was first introduced and used as a number and a digit in which civilisation?

Solution

The concept of zero as a number was developed in ancient India.

Answer

(c) Indian

---

3. The Roman numeral for 2367 is

Solution

$$2367 = 2000 + 300 + 60 + 7$$

2000 = MM

300 = CCC

60 = LX

7 = VII

Therefore,

$$2367 = MMCCCLXVII$$

Answer

(c) MMCCCLXVII

---

4. In the base-5 number system, the landmark numbers are

Solution

Landmark numbers in any base are powers of the base.

For base 5:

$$5^0, 5^1, 5^2, 5^3, \ldots$$

Answer

(c) Powers of 5

---

5. What did the Mayans use as the symbol for zero?

Solution

The Mayans used a shell-shaped symbol for zero.

Answer

(d) A seashell

---

6. Which ancient civilisation was associated with the sexagesimal number system?

Solution

Sexagesimal means base-60.

The Mesopotamians used a base-60 system.

Answer

(d) Mesopotamian

---

7. Who played an important role in spreading the Hindu numeral system to the Arab world?

Solution

Al-Khwarizmi wrote extensively about the Hindu numeral system.

Answer

(c) Al-Khwarizmi

---

8. Which statement is true about the Hindu number system?

Solution

The Hindu system:

- Uses place value.
- Uses zero.
- Uses ten digits.

Answer

(d) It uses 10 symbols including zero

B. Fill in the Blanks

1.

The earliest forms of counting were done using physical objects like **stones** or body parts.

2.

The digit 0 was first used in India and represented as a **dot** in the **Bakhshali** manuscript.

3.

The Mesopotamian number system had base **60** and that's why called a **sexagesimal** system.

4.

In the Roman numeral system, the symbol for 1000 is **M**.

5.

The Hindu number system uses **10** basic digits, including zero.

6.

In the Gumulgal number system, 2 was represented by **ukasar**.

7.

The place value system assigns a value to a digit based on its **position** in the number.

8.

A number system in which all landmark numbers are powers of n is called a **base-n number system**.

C. True / False

1.

The Roman numeral system is a place value system.

False

Roman numerals are non-positional.

2.

In base-5 system, valid digits are 0,1,2,3 and 5.

False

Valid digits are:

$$0,1,2,3,4$$

3.

Mesopotamians used a base-20 system.

False

They used base-60.

4.

Gumulgals used 'ras' for numbers greater than 6.

True

5.

The Mayan system was almost a base-20 system.

True

6.

The Chinese rod numeral T represents 7.

False

T represents 6.

7.

Mayan symbols are placed vertically from highest to lowest landmark numbers.

True

8.

Abacus was based on a base-5 number system.

False

It mainly follows the decimal system.

2. Match Column I with Column II

Column I	Column II
(a) Hindu-Arabic number system	(iii) 1, 10, 10^2, 10^3
(b) Base-2 system	(iv) 0, 1
(c) Roman numeral system	(ii) I, V, X, L, C, D, M
(d) Chinese rod system	(i) Zongs and Hengs

Answer

(a) → (iii)

(b) → (iv)

(c) → (ii)

(d) → (i)

E. Very Short Answer Questions

1. Why was zero considered a revolutionary invention in Mathematics?

Answer

Zero was a revolutionary invention because:

- It acts as a placeholder.
- It made place-value systems possible.
- It simplified calculations.
- It helped represent large numbers easily.

---

2. Define a place value system.

Answer

A place value system is a number system in which the value of a digit depends on its position in the number.

Example

In 456:

- 4 = 400
- 5 = 50
- 6 = 6

---

3. How did the Mesopotamians indicate a missing digit or empty space?

Answer

The Mesopotamians used a special placeholder symbol to indicate an empty place or missing digit.

---

4. Was the Mayan number system actually a base-20 system?

Answer

No.

It was an **almost base-20 system** because one place value was based on 18 × 20 instead of 20 × 20.

---

5. Define Zongs and Hengs.

Answer

The vertical and horizontal rods used in the Chinese rod numeral system are called **Zongs and Hengs**.

---

6. Write the first six landmark numbers of the Egyptian number system.

Solution

The Egyptian system is based on powers of 10.

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 100$$
$$10^3 = 1000$$
$$10^4 = 10000$$
$$10^5 = 100000$$

Answer

1, 10, 100, 1000, 10000, 100000

F. Short Answer Questions

1. Convert the following Roman numerals into Hindu-Arabic numerals.

(i) DXL

Solution

D = 500

XL = 50 − 10 = 40

$$500 + 40$$
$$= 540$$

Answer

$$\boxed{540}$$

(ii) CMXLIV

Solution

CM = 1000 − 100 = 900

XL = 50 − 10 = 40

IV = 5 − 1 = 4

$$900 + 40 + 4$$
$$= 944$$

Answer

$$\boxed{944}$$

(iii) LXXXIX

Solution

L = 50

XXX = 30

IX = 9

$$50 + 30 + 9$$
$$= 89$$

Answer

$$\boxed{89}$$

---

2. Convert the following Hindu-Arabic numerals into Roman numerals.

(i) 388

Solution

$$388 = 300 + 80 + 8$$

300 = CCC

80 = LXXX

8 = VIII

$$388 = CCC + LXXX + VIII$$

Answer

$$\boxed{CCCLXXXVIII}$$

(ii) 2071

Solution

$$2071 = 2000 + 70 + 1$$

2000 = MM

70 = LXX

1 = I

$$2071 = MMLXXI$$

Answer

$$\boxed{MMLXXI}$$

(iii) 56

Solution

$$56 = 50 + 6$$

50 = L

6 = VI

$$56 = LVI$$

Answer

$$\boxed{LVI}$$

---

3. Add the following Roman numerals

(i) XXX + LXX

Solution

XXX = 30

LXX = 70

$$30 + 70$$
$$= 100$$

100 = C

Answer

$$\boxed{C}$$

(ii) LX + XL

Solution

LX = 60

XL = 40

$$60 + 40$$
$$= 100$$

100 = C

Answer

$$\boxed{C}$$

(iii) CD + D

Solution

CD = 400

D = 500

$$400 + 500$$
$$= 900$$

900 = CM

Answer

$$\boxed{CM}$$

(iv) CCLX + DCCXL

Solution

CCLX

$$= 200 + 50 + 10$$
$$= 260$$

DCCXL

$$= 500 + 200 + 40$$
$$= 740$$

Now add:

$$260 + 740$$
$$= 1000$$

1000 = M

Answer

$\boxed{M}$

4. Use the Extended Gumulgal Number System (base-2 grouping) to perform the following operation without converting to Hindu numerals.

(i) (ukasar–urapon) + (ukasar–urapon)

Solution

ukasar–urapon + ukasar–urapon

= ukasar + ukasar + urapon + urapon

= ukasar + ukasar + ukasar

Answer

ukasar – ukasar – ukasar

(ii) (ukasar–ukasar–ukasar–urapon) – (ukasar)

Solution

Remove one ukasar from

ukasar–ukasar–ukasar–urapon

Result:

ukasar–ukasar–urapon

Answer

ukasar – ukasar – urapon

(iii) (ukasar–ukasar–urapon) × (urapon)

Solution

Multiplying by urapon (one) leaves the number unchanged.

Answer

ukasar – ukasar – urapon

(iv) (ukasar–ukasar–ukasar–ukasar–urapon) ÷ (ukasar–urapon)

Solution

$$9 \div 3 = 3$$

In Gumulgal form:

Answer

ukasar – urapon

---

5. Convert the Following Numerals into Egyptian Number System

(i) 426

$$426 = 400 + 20 + 6$$

Answer:

- 4 symbols of 100
- 2 symbols of 10
- 6 symbols of 1

(ii) 1032

$$1032 = 1000 + 30 + 2$$

Answer:

- 1 symbol of 1000
- 3 symbols of 10
- 2 symbols of 1

(iii) 5790

$$5790 = 5000 + 700 + 90$$

Answer:

- 5 symbols of 1000

- 7 symbols of 100
- 9 symbols of 10

---

6. Convert the Following Numerals into Mesopotamian System

(i) 43

Solution

$$43 = 4 \times 10 + 3$$

Answer

4 ten-symbols and 3 one-symbols

(ii) 219

Solution

$$219 = 3 \times 60 + 39$$

Answer

$$(3,39)_{60}$$

(iii) 1504

Solution

$$1504 = 25 \times 60 + 4$$

Answer

$$(25,4)_{60}$$

---

7. Convert the Following Mayan Numerals to Hindu-Arabic System

(i)

Given:

1 bar + 2 dots

$$5 + 2 = 7$$

Answer

$$\boxed{7}$$

(ii)

Top level:

3 dots + 2 bars

$$3 + 10 = 13$$

Bottom level:

Shell = 0

Value:

$$\begin{aligned} &13 \times 20 + 0 \\ &= 260 \end{aligned}$$

Answer

$$\boxed{260}$$

(iii)

Top level:

3 dots + 3 bars

$$3 + 15 = 18$$

Middle level:

1 dot

$$= 1$$

Bottom level:

Shell

$$= 0$$

Value:

$$\begin{aligned}&18 \times 400 + 1 \times 20 + 0\\&7200 + 20\\&7220\end{aligned}$$

Answer

$$\boxed{7220}$$

---

8. Convert the Following Numerals into Chinese Rod Numerals

(i) 84

Solution

$$84 = 8 \times 10 + 4$$

Tens place → horizontal rods for 8

Ones place → vertical rods for 4

Answer

Chinese rod numeral for **84**

(ii) 307

Solution

$$307 = 3 \times 100 + 0 \times 10 + 7$$

Hundreds → 3 rods

Tens → empty place

Ones → 7 rods

Answer

Chinese rod numeral for **307**

(iii) 1926

Solution

$$1926 = 1 \times 1000 + 9 \times 100 + 2 \times 10 + 6$$

Thousands → 1

Hundreds → 9

Tens → 2

Ones → 6

using alternate vertical and horizontal rod arrangements.

Answer

Chinese rod numeral for **1926**.

G. Long Answer Type Questions

---

1. Describe how the Chinese rod numeral system worked. How were different digits and place values represented using rods? Provide examples to illustrate.

Answer

The Chinese rod numeral system was an ancient numeral system used in China for counting and calculations.

Working of the System

- Numbers were represented using small rods.
- Rods were arranged vertically and horizontally.

- Vertical rods were called **Zongs**.
- Horizontal rods were called **Hengs**.
- The direction of rods changed from one place value to the next to avoid confusion.

Representation of Place Values

Place Value	Rod Type
Ones	Vertical
Tens	Horizontal
Hundreds	Vertical
Thousands	Horizontal

Example

For the number 243:

$$243 = 2 \times 100 + 4 \times 10 + 3$$

- 2 hundreds represented using vertical rods.
- 4 tens represented using horizontal rods.
- 3 ones represented using vertical rods.

Thus the position of rods indicated place value.

Conclusion

The Chinese rod numeral system was an early place-value system that made calculations easier than many other ancient numeral systems.

---

2. (i) Explain the evolution of the idea of counting, beginning from the Stone Age to the invention of the Hindu number system.

Answer

The idea of counting developed gradually over thousands of years.

Stone Age

- Early humans counted using fingers and toes.

- They used stones, pebbles, shells and sticks.

Tally Marks

- Marks were cut on bones and wooden sticks.
- Each mark represented one object.

Ancient Civilisations

- Egyptians developed a base-10 system.
- Mesopotamians developed a base-60 system.
- Romans developed Roman numerals.
- Mayans developed an almost base-20 system.

Hindu Number System

Indian mathematicians developed:

- Digits 0–9
- Place value system
- Zero as a number

This became the modern Hindu-Arabic number system.

(ii) How did early humans keep track of quantity?

Answer

Early humans kept track of quantity by using:

- Fingers and toes
- Stones and pebbles
- Shells
- Knots on ropes
- Tally marks on bones and sticks

These methods helped them record and compare quantities.

(iii) What methods did they use for representation before written numerals?

Answer

Before written numerals, people used:

1. Fingers and body parts
2. Pebbles and stones
3. Tally marks
4. Knotted ropes
5. Shells and seeds

These objects represented quantities and helped in counting.

---

3. (i) What is a positional number system?

Answer

A positional number system is a number system in which the value of a digit depends on its position.

Example

In the number:

$$
\begin{aligned}
&352 \\
&3 = 300 \\
&5 = 50 \\
&2 = 2
\end{aligned}
$$

Thus the same digit can have different values depending on its place.

(ii) Explain the concept using the Hindu number system with examples.

Answer

The Hindu number system is a positional system with base 10.

It uses ten digits:

$$0,1,2,3,4,5,6,7,8,9$$

Example

Consider:

$$4567$$

Expanded form:

$$4567 = (4 \times 1000) + (5 \times 100) + (6 \times 10) + (7 \times 1)$$
$$= 4000 + 500 + 60 + 7$$

Each digit gets its value from its position.

Hence, the Hindu number system is a positional number system.

G.4 Add the Following Egyptian Numerals

(i)

Given:

(3 spirals + 2 tens + 2 ones) + (2 tens + 4 ones)

Step 1: Convert each numeral

First numeral

$$= 3 \times 100 + 2 \times 10 + 2$$
$$= 300 + 20 + 2$$
$$= 322$$

Second numeral

$$= 2 \times 10 + 4$$
$$= 20 + 4$$
$$= 24$$

Step 2: Add

$$322 + 24$$
$$= 346$$

Step 3: Write in Egyptian form

$$346 = 300 + 40 + 6$$

Answer

$$\boxed{346}$$

(Egyptian form: 3 spirals, 4 tens, 6 ones)

(ii)

Given:

(Lotus + 1 ten + 2 ones) + (Finger + 1 ten + 2 ones)

Step 1: Convert each numeral

First numeral

$$1000 + 10 + 2$$
$$= 1012$$

Second numeral

$$10000 + 10 + 2$$
$$= 10012$$

Step 2: Add

$$1012 + 10012$$
$$= 11024$$

Step 3: Write in Egyptian form

$$11024 = 10000 + 1000 + 20 + 4$$

Answer

$$\boxed{11024}$$

(Egyptian form: 1 finger, 1 lotus, 2 tens, 4 ones)

(iii)

Given:

(3 spirals + 5 tens + 5 ones) + (6 spirals + 4 tens + 5 ones)

Step 1: Convert each numeral

First numeral

$$300 + 50 + 5 \\ = 355$$

Second numeral

$$600 + 40 + 5 \\ = 645$$

Step 2: Add

$$355 + 645 \\ = 1000$$

Step 3: Write in Egyptian form

$$1000$$

Answer

$$\boxed{1000}$$

(Egyptian form: 1 lotus symbol)

G.5 Find the Following Products

Important Property

Multiplying by ∩ (10) shifts every Egyptian symbol to the next higher landmark.

Examples:

$$1 \times 10 = 10 \\ 10 \times 10 = 100 \\ 100 \times 10 = 1000$$

$$1000 \times 10 = 10000$$

(i)

Given:

(3 spirals + 2 tens + 3 ones) × ∩

Step 1: Convert numeral

$$300 + 20 + 3$$
$$= 323$$

Step 2: Multiply

$$323 \times 10$$
$$= 3230$$

Step 3: Egyptian form

$$3230 = 3000 + 200 + 30$$

Answer

$$\boxed{3230}$$

(Egyptian form: 3 lotus, 2 spirals, 3 tens)

(ii)

Given:

(2 fingers + 2 lotus + 1 ten + 3 ones) × ∩

Step 1: Convert numeral

$$20000 + 2000 + 10 + 3$$
$$= 22013$$

Step 2: Multiply

$$22013 \times 10$$
$$= 220130$$

Step 3: Egyptian form

$$220130 = 22 \times 10000 + 100 + 30$$

Answer

$$\boxed{220130}$$

(Egyptian form: 22 fingers, 1 spiral, 3 tens)

(iii)

Given:

Lotus × ∩

Step 1

Lotus

$$= 1000$$

Step 2

$$1000 \times 10$$
$$= 10000$$

Answer

$$\boxed{10000}$$

(Egyptian form: 1 finger symbol)

Competency-Based Questions

A. Assertion–Reason Questions

1.

Assertion (A): Zero was essential for developing a place-value number system.

Reason (R): Zero acts as a placeholder to maintain the correct position of digits.

Explanation

- Assertion is true.
- Reason is true.
- Reason correctly explains the assertion.

Answer

(a) Both A and R are true and R is the correct explanation of A.

2.

Assertion (A): The Gumulgal system is not a positional number system.

Reason (R): It used the same digits repeatedly based on place values and powers of two.

Explanation

- Assertion is true.
- Reason is false because Gumulgal does not use place values.

Answer

(c) A is true but R is false.

3.

Assertion (A): The number 49 is written as XLIX in Roman numerals.

Reason (R): In Roman numerals, when a smaller value precedes a larger one, it is subtracted.

Explanation

XL = 50 − 10 = 40

IX = 10 − 1 = 9

$$40 + 9 = 49$$

Both statements are true and R explains A.

Answer

(a) Both A and R are true and R is the correct explanation of A.

4.

Assertion (A): In Mesopotamian system, 183 is written as 3×60 + 3.

Reason (R): Mesopotamians used a base-60 system.

Explanation

$$183 = 3 \times 60 + 3$$

Both statements are true.

Answer

(a) Both A and R are true and R is the correct explanation of A.

B. Case Study Based Questions

1. Egyptian Reeds Problem

A high priest prepares 432 bundles of reeds and supplies them equally to 3 temples.

(i) Write the Egyptian representation of 432.

Solution

$$432 = 400 + 30 + 2$$

Thus:

- 4 symbols of 100
- 3 symbols of 10
- 2 symbols of 1

Answer

Egyptian numeral for 432 consists of:

4 hundred symbols, 3 ten symbols and 2 one symbols.

(ii) Use distributive law to compute total bundles supplied to all three temples.

Solution

Each temple receives 432 bundles.

Total:

$$432 \times 3$$

Using distributive law:

$$\begin{aligned}&(400 + 30 + 2) \times 3\\&= 400 \times 3 + 30 \times 3 + 2 \times 3\\&= 1200 + 90 + 6\\&= 1296\end{aligned}$$

Answer

$$\boxed{1296}$$

(iii) How many total bundles of reeds does he supply in Hindu-Arabic numerals?

Answer

$$\boxed{1296}$$

---

2. Mayan Priest Problem

A Mayan priest records 841 days.

(i) Record these days in Mayan numerals.

Solution

$$841 = 2 \times 400 + 2 \times 20 + 1$$

Thus:

Top level = 2 dots

Middle level = 2 dots

Bottom level = 1 dot

Answer

Mayan representation:

- Top: ••
- Middle: ••
- Bottom: •

(ii) Determine how many more days are needed until the 1000th day celebration.

Solution

$$\begin{aligned} &1000 - 841 \\ &= 159 \end{aligned}$$

Answer

$$\boxed{159 \text{ days}}$$

(iii) Write one advantage and one disadvantage of this system.

Advantage

Large numbers can be represented using fewer symbols.

Disadvantage

The system is difficult to learn and perform calculations with.

Maths Booster

1. Decode the Roman Code

A three-digit number uses exactly 12 Roman symbols.

Find the smallest such number.

Solution

Consider:

$$888$$

Roman numeral:

$$888 = DCCCLXXXVIII$$

Count symbols:

D = 1

CCC = 3

L = 1

XXX = 3

VIII = 4

Total:

$$1 + 3 + 1 + 3 + 4 = 12$$

Answer

$$\boxed{888}$$

Roman numeral:

$$\boxed{DCCCLXXXVIII}$$

---

2. Mayan vs Egyptian Race

Represent 720.

Egyptian System

$$720 = 700 + 20$$

Symbols required:

- 7 hundred symbols
- 2 ten symbols

Total:

9 symbols

Mayan System

$$720 = 1 \times 400 + 16 \times 20 + 0$$

Top level:

1 dot = 1 symbol

Middle level:

16 = 3 bars + 1 dot

= 4 symbols

Bottom level:

Shell = 1 symbol

Total:

$$1 + 4 + 1 = 6$$

Conclusion

$$6 < 9$$

Therefore,

Answer

Reema (Mayan system) uses fewer symbols than Riya (Egyptian system) to write 720.

Chapter 4: Quadrilaterals

NCERT CORNER

IN TEXT 1

Question

In the earlier definition, we stated that a rectangle has

(a) opposite sides of equal length and

(b) all angles equal to 90°.

Would we be wrong if we just define a rectangle as a quadrilateral in which all the angles are 90°?

Solution

Suppose a quadrilateral has four right angles.

Then:

$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$

Such a quadrilateral is always a rectangle.

Also, in such a figure, opposite sides automatically become equal and parallel.

Therefore, mentioning "opposite sides are equal" is unnecessary.

Answer

No, we would not be wrong. A rectangle can simply be defined as a quadrilateral whose four angles are 90°. The property of opposite sides being equal follows automatically.

IN TEXT 2

Question

If you think that this definition is incomplete, try constructing a quadrilateral in which all the angles are 90° but the opposite sides are not equal.

Solution

Try drawing such a figure.

Whenever all four angles are 90°, the figure formed is always a rectangle.

In every rectangle:

- Opposite sides are equal.
- Opposite sides are parallel.

Hence such a quadrilateral cannot be constructed.

Answer

It is not possible to construct a quadrilateral having all four angles equal to $90^\circ$and opposite sides unequal. Such a figure will always be a rectangle.

IN TEXT

Question

If $\triangle BAD \cong \triangle DCB$, then is it wrong to write $\triangle BAD \cong \triangle CDB$? Why?

Solution

In congruent triangles, the order of vertices is important.

Given:

$$\triangle BAD \cong \triangle DCB$$

Corresponding vertices are:

$$\begin{gathered} B \leftrightarrow D \\ A \leftrightarrow C \\ D \leftrightarrow B \end{gathered}$$

If we write

$$\triangle BAD \cong \triangle CDB$$

the corresponding vertices are changed incorrectly.

Therefore, the matching of sides and angles will not remain correct.

Answer

Yes, it is wrong. In congruent triangles, the order of vertices must follow the correspondence of equal sides and equal angles. Therefore, $\triangle BAD \cong \triangle DCB$is correct, but $\triangle BAD \cong \triangle CDB$is not.

IN TEXT

Question

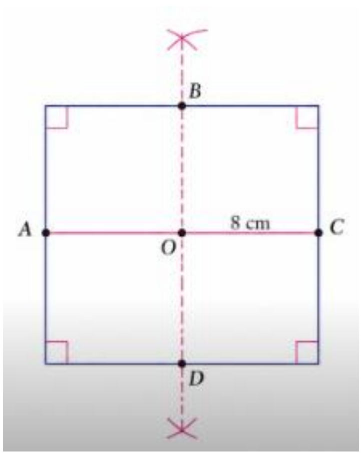
Using this fact, construct a square with a diagonal of length 8 cm.

Construction

Given: Diagonal = 8 cm

Steps

1. Draw a line segment $AC = 8$cm.
2. Find the midpoint $O$of AC.
3. Draw a perpendicular line through O.
4. With centre $O$and radius 4cm, cut the perpendicular at points $B$and D.
5. Join $AB, BC, CD,$and DA.



Verification

- $AC = BD = 8$cm

- Diagonals bisect each other at right angles.
- All sides become equal.

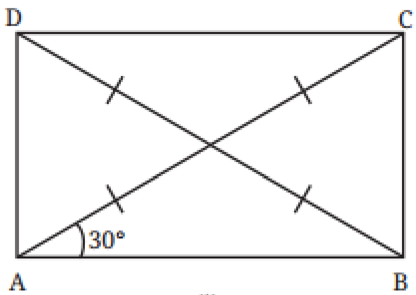
Hence $ABCD$is a square.

Answer

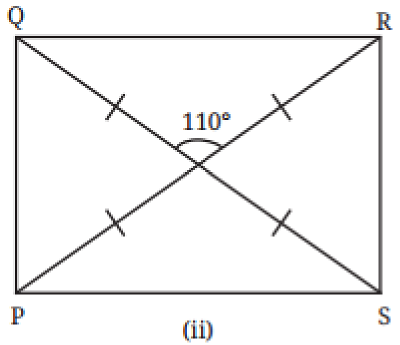
The required square $ABCD$is obtained by drawing diagonal $AC = 8$cm, constructing its perpendicular bisector, locating $B$and Dat a distance of 4 cm from the midpoint, and joining the vertices.

Figure It Out

Question 1.
Find all the other angles inside the following rectangles.

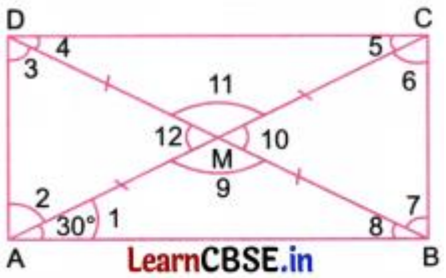


(i)



(ii)

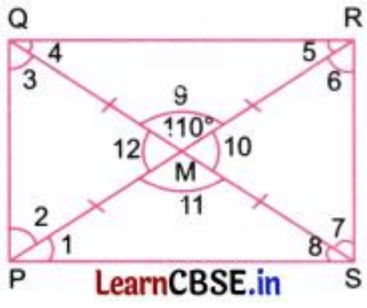
Solution:
(i) The given rectangle is ABCD.



We have ∠1 = 30°
∠1 + ∠2 = 90°

$\therefore \angle 2 = 90^\circ - \angle 1 = 90^\circ - 30^\circ = 60^\circ$.
MD = MA
$\Rightarrow \angle 3 = \angle 2 = 60^\circ$
$\angle 3 + \angle Z4 = 90^\circ$
$\therefore \angle 4 = 90^\circ - \angle 3 = 90^\circ - 60^\circ = 30^\circ$
MC = MD
$\Rightarrow \angle 5 = \angle 4 = 30^\circ$
$\angle 5 + \angle 6 = 90^\circ$
$\therefore \angle 6 = 90^\circ - \angle 5 = 90^\circ - 30^\circ = 60^\circ$
MB = MC
$\Rightarrow \angle 7 = \angle 6 = 60^\circ$
MB = MA
$\Rightarrow \angle 8 = \angle 1 = 30^\circ$
In ΔAMB, we have
$\angle 1 + \angle 9 + \angle 8 = 180^\circ$.
$\therefore 30^\circ + \angle 9 + 30^\circ = 180^\circ$
$\therefore \angle 9 = 180^\circ - 60^\circ = 120^\circ$
$\angle 11 = \angle 9 = 120^\circ$ (Vertically opposite angles)
$\angle 9 + \angle 10 = 180^\circ$ (Linear angles)
$\therefore \angle 10 = 180^\circ - 120^\circ = 60^\circ$
$\angle 12 = \angle 10 = 60^\circ$ (Vertically opposite angles)
$\therefore \angle 2 = 60^\circ, \angle 3 = 60^\circ, \angle 4 = 30^\circ, \angle 5 = 30^\circ, \angle 6 = 60^\circ, \angle 7 = 60^\circ, \angle 8 = 30^\circ, \angle 9 = 120^\circ, \angle 10 = 60^\circ, \angle 11 = 120^\circ$ and $\angle 12 = 60^\circ$.

(ii) The given rectangle is PSRQ.

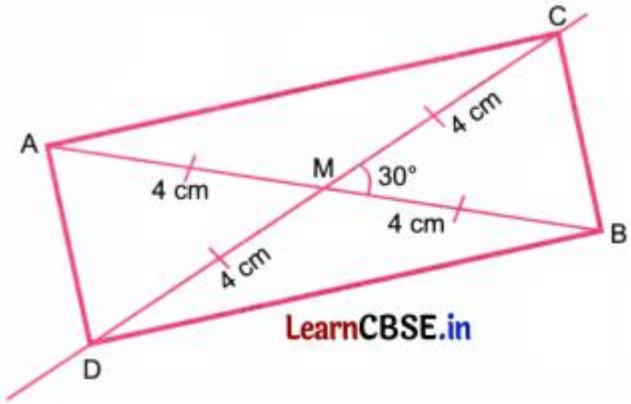


We have $\angle 9 = 110^\circ$.
$\angle 11 = \angle 9 = 110^\circ$ (Vertically opposite angles)
$\angle 9 + \angle 10 = 180^\circ$ (Linear angles)
$\therefore \angle 10 = 180^\circ - 110^\circ = 70^\circ$
$\angle 12 = \angle 10 = 70^\circ$ (Vertically opposite angles)
MP = MS
$\Rightarrow \angle 1 = \angle 8$
In ΔPMS, we have

$\angle 1 + \angle 11 + \angle 8 = 180°$.
$\Rightarrow \angle 1 + 110 + \angle 1 = 180°$
$\Rightarrow 2\angle 1 = 180° – 110$
$\Rightarrow 2\angle 1 = 70°$
$\Rightarrow \angle 1 = 35°$
$\therefore \angle 8$ is also 35°.
$\therefore \angle 1 + \angle 2 = 90°$
$\Rightarrow \angle 2 = 90° – \angle 1$
$\Rightarrow \angle 2 = 90° – 35°$
$\Rightarrow \angle 2 = 55°$
MQ = MP
$\Rightarrow \angle 3 = \angle 2 = 55°$
$\therefore \angle 3 + \angle 4 = 90°$
$\Rightarrow \angle 4 = 90° – \angle 3$
$\Rightarrow \angle 4 = 90° – 55°$
$\Rightarrow \angle 4 = 35°$
MR = MQ
$\Rightarrow \angle 5 = \angle 4 = 35°$
$\therefore \angle 5 + \angle 6 = 90°$
$\Rightarrow \angle 6 = 90° – \angle 5$
$\Rightarrow \angle 6 = 90° – 35°$
$\Rightarrow \angle 6 = 55°$
MS = MR
$\Rightarrow \angle 7 = \angle 6 = 55°$
$\therefore \angle 1 = 35°, \angle 2 = 55°, \angle 3 = 55°, \angle 4 = 35°, \angle 5 = 35°, \angle 6 = 55°, \angle 7 = 55°, \angle 8 = 35°, \angle 10 = 70°, \angle 11 = 110°$ and $\angle 12 = 70°$.

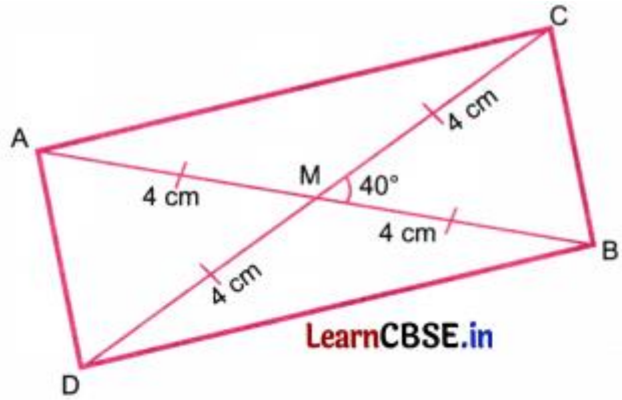
Question 2.
Draw a quadrilateral whose diagonals have equal lengths of 8 cm that bisect each other, and intersect at an angle of:
(i) 30°
(ii) 40°
(iii) 90°
(iv) 140°
Solution:
(i) Draw a line AB equal to 8 cm.
Take point M on AB such that AM = BM = 4 cm.
Using a protractor, draw an angle of 30° at M on MB.

On this line, take points C and D such that MC = MD = 4 cm.
Join AD, DB, BC, and CA.
ABCD is the required quadrilateral.



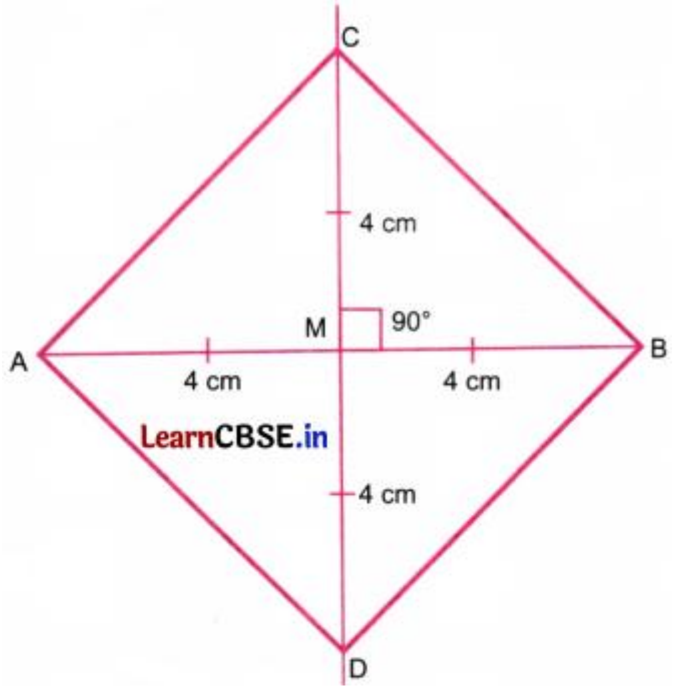
Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

(ii) Draw a line AB equal to 8 cm.
Take point M on AB such that AM = BM = 4 cm.
Using a protractor, draw an angle of 40° at M on MB.
On this line, take points C and D such that MC = MD = 4 cm.
Join AD, DB, BC, and CA.
ABCD is the required quadrilateral.



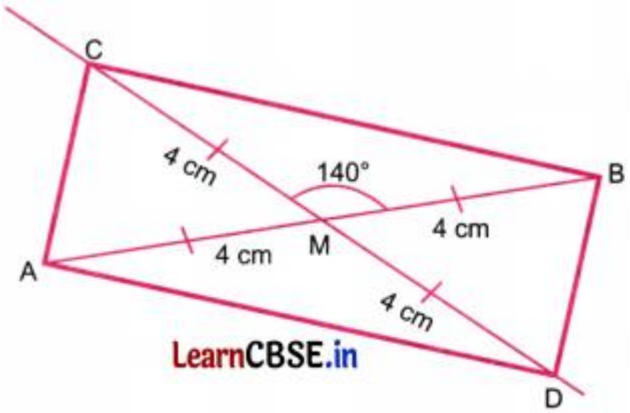
Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

(iii) Draw a line AB equal to 8 cm.
Take a point M on AB such that AM = BM = 4 cm.
Using a protractor, draw an angle of 90° at M on MB.
On this line, take points C and D such that MC = MD = 4 cm.
Join AD, DB, BC, and CA.
ACBD is the required square.



Since diagonals AB and CD are equal and are bisecting each other at M, and also the diagonals are perpendicular to each other, ACBD is a square.

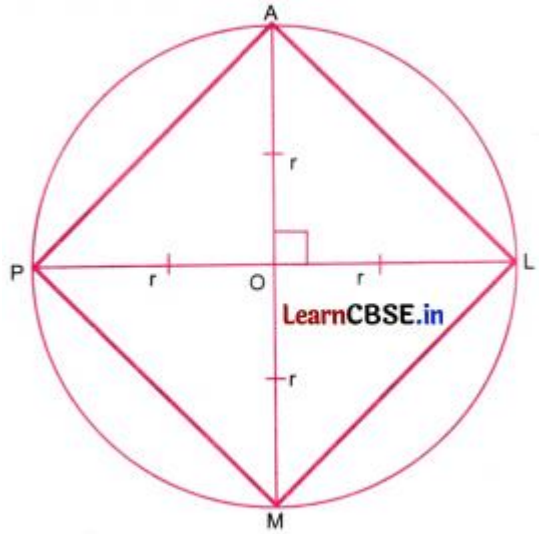
(iv) Draw a line AB equal to 8 cm.
Take a point M on AB such that AM = BM = 4 cm.
Using a protractor, draw an angle of 140° at M on MB.
On this line, take points C and D such that MC = MD = 4 cm.
Join AD, DB, BC, and CA.
ACBD is the required quadrilateral.



Since diagonals AB and CD are equal and are bisecting each other at M, ACBD is a rectangle.

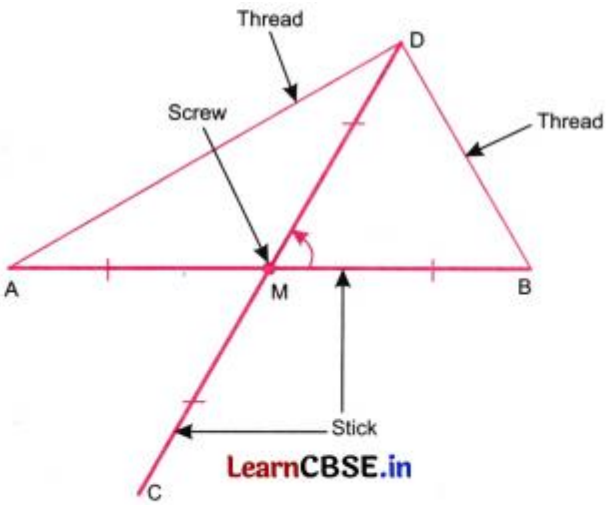
Question 3.
Consider a circle with centre O. Line segments PL and AM are two perpendicular diameters of the circle. What is the figure APML? Reason and/or

experiment to figure this out.
Solution:
In the figure, PL and AM are two perpendicular diameters of the circle.
Let r be the radius of the circle.

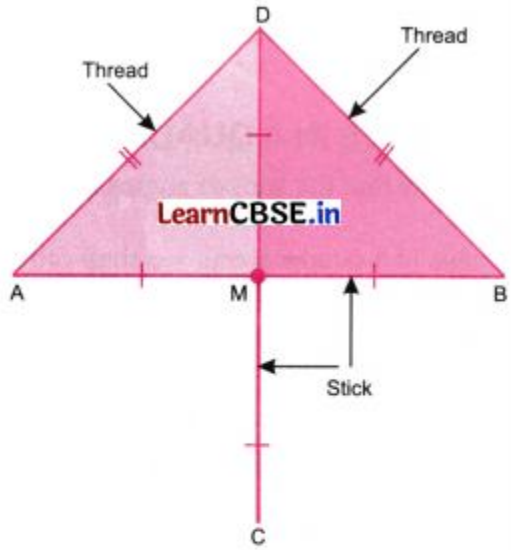


∴ PL = PO + OL
= r + r
= 2r
and AM = AO + OM
= r + r
= 2r
∴ PL = AM
∴ In the quadrilateral APML, diagonals PL and AM are equal and are perpendicular to each other.
Also, OP = OA = OL = OM = r
∴ Diameters PL and AM bisect each other at O.
∴ Quadrilateral APML is a square.

Question 4.
We have seen how to get 90° using paper folding. Now, suppose we do not have any paper but two sticks of equal length and a thread. How do we make an exact 90° angle using these?
Solution:
Let AB and CD be two sticks of equal length, say 6 cm.
Mark the midpoints of the sticks using a ruler.
Fix a screw to the sticks at their midpoints.
Using a thread, measure distances AD and BD.
Keep on moving the sticks about the screw, so that the distances AD and BD are equal.



In this position, fix the sticks by tightening the screw.
The new positions of the sticks are shown in the figure.
Tie pieces of thread along AD and BD.



Consider ΔAMD and ΔBMD.
We have AM = BM, AD = BD
and MD is common
∴ By the SSS condition,
ΔAMD and ΔBMD are congruent.
∴ $\angle AMD = \angle BMD$
Also $\angle AMD + \angle BMD = 180°$ (Linear angles)
∴ $\angle AMD + \angle AMD = 180°$
⇒ $2\angle AMD = 180°$
⇒ $\angle AMD = 90°$
∴ $\angle AMD = \angle BMD = 90°$
∴ Angle between the sticks is 90°.

Question 5.
We saw that one of the properties of a rectangle is that its opposite sides are parallel. Can this be chosen as a definition of a rectangle? In other words, is every quadrilateral that has opposite sides parallel and equal a rectangle?
Solution:
Let ABCD be a quadrilateral in which opposite sides are parallel and equal.
Here AB || DC and AD || BC.
Also, AB = DC and AD = BC.
In the quadrilateral ABCD, opposite sides are equal.
For ABCD to be a rectangle, we require each angle to be 90°.
Given information AB || DC and AD || BC can not help us to prove that each angle of ABCD is 90°.

∴ ABCD may not be a rectangle.
∴ A rectangle can not be defined as a quadrilateral with equal and parallel opposite sides.

IN-TEXT QUESTION

In the given parallelogram $ABCD$,

- $AB = 4\text{cm}$
- $AD = 5\text{cm}$
- $\angle A = 30^\circ$

Find the remaining angles and side lengths.

Solution

Property of a parallelogram:

1. Opposite sides are equal.
2. Opposite angles are equal.
3. Adjacent angles are supplementary.

Given:

$$\angle A = 30^\circ$$

Therefore,

$$\angle C = \angle A = 30°$$

Also,

$$\angle B = 180° - 30°$$
$$= 150°$$
$$\angle D = 150°$$

Now,

$$AB = CD = 4 \text{ cm}$$
$$AD = BC = 5 \text{ cm}$$

Answer

$$\angle A = \angle C = 30°$$
$$\angle B = \angle D = 150°$$
$$AB = CD = 4 \text{ cm}$$
$$AD = BC = 5 \text{ cm}$$

IN-TEXT QUESTION

If $\triangle ABD \cong \triangle CDB$, then is it wrong to write $\triangle ABD \cong \triangle CBD$? Why?

Answer

Yes, it is wrong.

In congruent triangles, the order of vertices must show corresponding vertices correctly.

Given:

$$\triangle ABD \cong \triangle CDB$$

Correspondence:

$$A \leftrightarrow C$$

$$B \leftrightarrow D$$
$$D \leftrightarrow B$$

Writing

$$\triangle ABD \cong \triangle CBD$$

changes the correspondence of vertices.

Hence it is incorrect.

IN-TEXT QUESTION

If $\triangle AOE \cong \triangle YOS$, then is it wrong to write $\triangle AOE \cong \triangle SOY$? Why?

Answer

Yes.

The order of letters must follow corresponding vertices.

Given

$$\triangle AOE \cong \triangle YOS$$

Corresponding vertices are

$$A \leftrightarrow Y$$
$$O \leftrightarrow O$$
$$E \leftrightarrow S$$

But in

$$\triangle SOY$$

the correspondence changes.

Hence it is wrong.

IN-TEXT QUESTION

Do the diagonals of a parallelogram intersect at a particular angle?

Answer

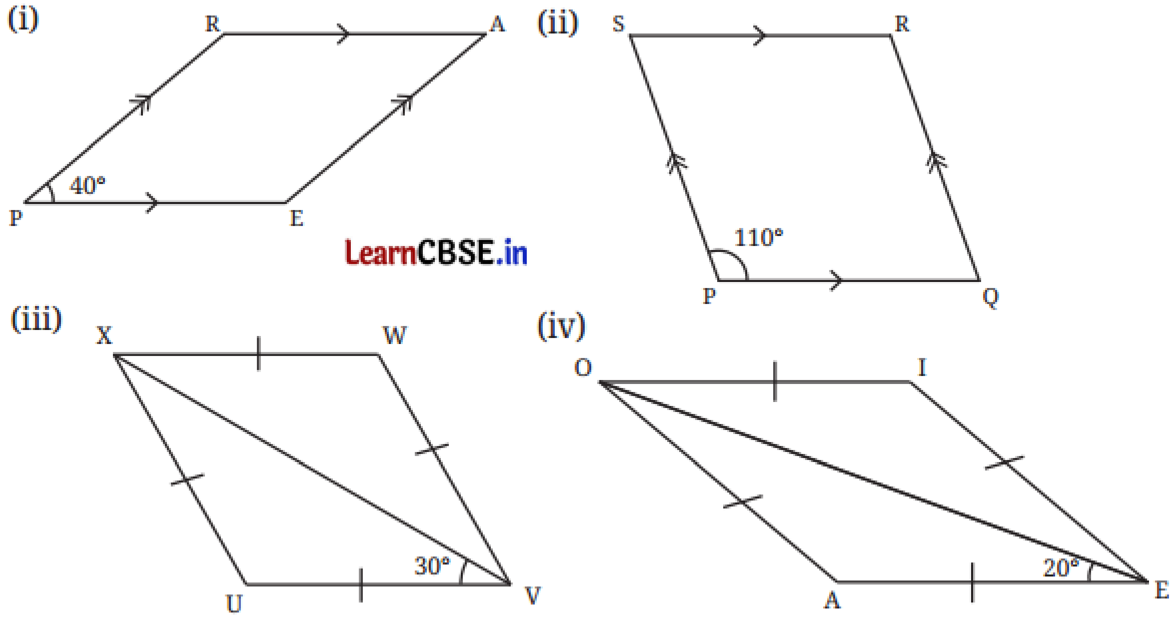
No.

The diagonals of a parallelogram bisect each other, but they do not necessarily intersect at 90°.

Only in special cases such as a rhombus or square do the diagonals intersect at right angles.

Figure It Out

Question 1.
Find the remaining angles in the following quadrilaterals.

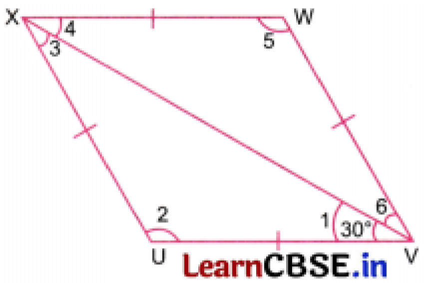


Solution:
(i) Since opposite sides of the quadrilateral PEAR are parallel, this is a parallelogram.
In the given parallelogram, PE is a transversal to the parallel lines PR and EA.
∠RPE and ∠AEP are the internal angles on the same side of parallel lines.
∴ ∠RPE + ∠AEP = 180°
⇒ 40° + ∠AEP = 180°
⇒ ∠AEP = 180° – 40° = 140°
We know that opposite angles of a parallelogram are equal.
∴ ∠EAR = ∠EPR = 40° and ∠ARP = ∠AEP = 140°.

(ii) Since opposite sides of the quadrilateral PQRS are parallel, this is a parallelogram.
In the given parallelogram, PQ is a transversal to the parallel lines PS and QR.

∠SPQ and ∠RQP are the internal angles on the same sides of the parallel lines.
∴ ∠SPQ + ∠RQP = 180°
⇒ 110° + ∠RQP = 180°
⇒ ∠RQP = 180° – 110° = 70°
We know that the opposite angles of a parallelogram are equal.
∴ ∠QRS = ∠QPS = 110° and ∠PSR = ∠RQP = 70°.

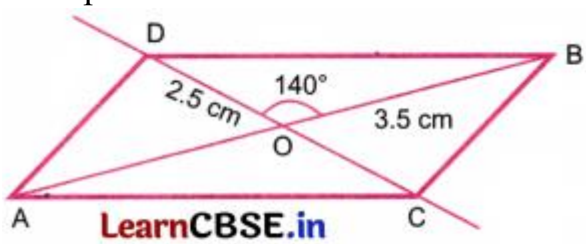
(iii) Here, quadrilateral UVWX is a rhombus, because all sides are equal.
We have ∠1 = 30°.
In a rhombus, diagonals bisect the angles of the rhombus.
∴ ∠6 = 30°
In ΔUVX, ∠1 = ∠3 (∵ UV = UX)
∴ ∠3 = 30°
∴ ∠4 = 30°



In ΔUVX, ∠1 + ∠2 + ∠3 = 180°.
∴ 30° + ∠2 + 30° = 180°
⇒ ∠2 = 180° – 60° = 120°
⇒ ∠5 = ∠2 = 120° (∵ Opposite angles are equal)
∴ ∠2 = 120°, ∠3 = 30°, ∠4 = 30°, ∠5 = 120° and ∠6 = 30°.

(iv) Please try yourself.

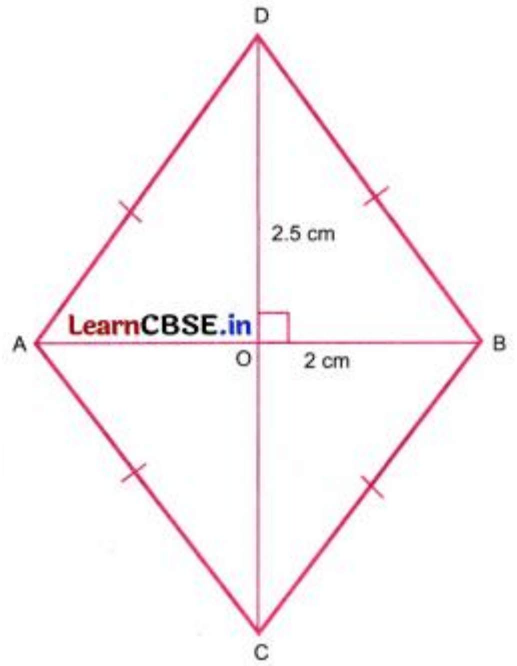
Question 2.
Using the diagonal properties, construct a parallelogram whose diagonals are of lengths 7 cm and 5 cm, and intersect at an angle of 140°.
Solution:
Draw a line AB equal to 7 cm.
Take point O on AB such that AO = OB = 3.5 cm.



On OB, draw an angle of 140° at O.
Take points C and D on the line of angle so that OC = OD = 2.5 cm.

∴ CD = 5 cm, and O is the midpoint of AB and AC.
Join AC, CB, BD, and DA.
ACBD is a quadrilateral, and its diagonals AB and CD bisect at O.
∴ ACBD is the required parallelogram.

Question 3.
Using the diagonal properties, construct a rhombus whose diagonals are of lengths 4 cm and 5 cm.
Solution:
Draw a line AB equal to 4 cm.
Take a point O on AB such that AO = OB = 2 cm.
On AB, draw a line perpendicular to it and passing through O.
Take points C and D on this perpendicular so that OC = OD = 2.5 cm.



∴ CD = 5 cm, and O is the midpoint of AB and CD.
Join AC, CB, BD, and DA.
ACBD is a quadrilateral, and its diagonals AB and CD are bisecting at O and are also perpendicular to each other.
∴ ACBD is the required rhombus.

GEOBOARD ACTIVITY

(i) Place two rubber bands perpendicular to each other, forming diagonals of equal lengths. Join the ends.

Answer

The quadrilateral formed is a **Square**.

Justification

- The diagonals are **equal**.
- The diagonals are **perpendicular** to each other.
- The diagonals bisect each other.

A quadrilateral whose diagonals are equal, bisect each other, and intersect at right angles is a **square**.

(ii) Extend one of the diagonals on both sides by 2 cm.

Answer

The quadrilateral formed is a **Rhombus**.

Justification

- The diagonals remain perpendicular.
- One diagonal becomes longer than the other.
- The diagonals still bisect each other.
- The diagonals are no longer equal.

A quadrilateral whose diagonals bisect each other at right angles but are unequal is a **rhombus**.

JOINING TRIANGLES

1(i)

Take two equilateral triangles of side 8 cm.

Question

Can you join them to get a quadrilateral?

Answer

Yes.

If the two equilateral triangles are joined along one side, a quadrilateral is formed.

1(ii)

Question

What type of quadrilateral is formed?

Solution

All four outer sides are marked:

$$8 \text{ cm},\ 8 \text{ cm},\ 8 \text{ cm},\ 8 \text{ cm}$$

Hence all sides are equal.

Answer

The quadrilateral formed is a **Rhombus**.

Justification

A rhombus is a quadrilateral with all four sides equal.

Since each side measures 8 cm, the figure is a rhombus.

2(i)

Take two isosceles triangles of sides

$$8 \text{ cm},\ 8 \text{ cm},\ 6 \text{ cm}$$

Question

What are the different ways they can be joined to get a quadrilateral?

Answer

The triangles can be joined:

1. Along their **6 cm sides**.
2. Along one of their **8 cm sides**.

Both arrangements produce quadrilaterals.

2(ii)(a)

Identify the quadrilateral.

In the figure:

All four outer sides are

$$8 \text{ cm}$$

Answer

Rhombus

Justification

All four sides are equal.

Therefore, the quadrilateral is a rhombus.

2(ii)(b)

Identify the quadrilateral.

Given:

- Top side = 6 cm
- Bottom side = 6 cm
- Left side = 8 cm
- Right side = 8 cm

Thus,

$$\text{Opposite sides are equal}$$

Answer

Parallelogram

Justification

A parallelogram has both pairs of opposite sides equal.

Here,

$$AB = CD = 6 \text{ cm}$$

and

$$BC = AD = 8 \text{ cm}$$

Hence the quadrilateral is a parallelogram.

Activity 2

(i) Take two cardboard cutouts of a scalene triangle with sides 6 cm, 9 cm and 12 cm.

Question

What are the different ways they can be joined to get a quadrilateral?

Solution

The two congruent scalene triangles can be joined along any one of their equal corresponding sides:

Case 1

Join along the **6 cm side**

Outer sides:

$$9 \text{ cm},\ 12 \text{ cm},\ 9 \text{ cm},\ 12 \text{ cm}$$

Quadrilateral formed has opposite sides equal.

Case 2

Join along the **9 cm side**

Outer sides:

$$6 \text{ cm},\ 12 \text{ cm},\ 6 \text{ cm},\ 12 \text{ cm}$$

Quadrilateral formed has opposite sides equal.

Case 3

Join along the **12 cm side**

Outer sides:

$$6 \text{ cm},\ 9 \text{ cm},\ 6 \text{ cm},\ 9 \text{ cm}$$

Quadrilateral formed has opposite sides equal.

Answer

The triangles can be joined in **three different ways**:

1. Along the 6 cm side
2. Along the 9 cm side
3. Along the 12 cm side

(ii) Are you able to identify the different quadrilaterals obtained by joining the triangles? Justify your answer.

Solution

When two congruent triangles are joined along a corresponding side:

- The common side becomes a diagonal.
- The remaining opposite sides become equal.
- The opposite sides are parallel.

Thus each figure obtained is a **parallelogram**.

Justification

When joined along 6 cm side

Outer sides:

$$9,\ 12,\ 9,\ 12$$

Opposite sides are equal.

Therefore, it is a **parallelogram**.

When joined along 9 cm side

Outer sides:

$$6,\ 12,\ 6,\ 12$$

Opposite sides are equal.

Therefore, it is a **parallelogram**.

When joined along 12 cm side

Outer sides:

$$6,\ 9,\ 6,\ 9$$

Opposite sides are equal.

Therefore, it is a **parallelogram**.

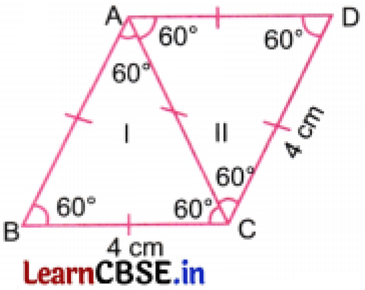
Answer

The three quadrilaterals obtained are **parallelograms**.

This is because in each case the opposite sides are equal (and parallel), which is the defining property of a parallelogram.

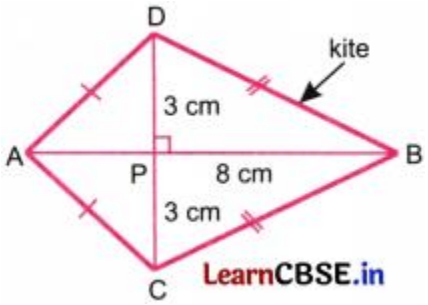
Figure It Out

Question 1.
Find all the sides and the angles of the quadrilateral obtained by joining two equilateral triangles with sides 4 cm.
Solution:
Let two equilateral triangles of sides 4 cm be joined as shown below.



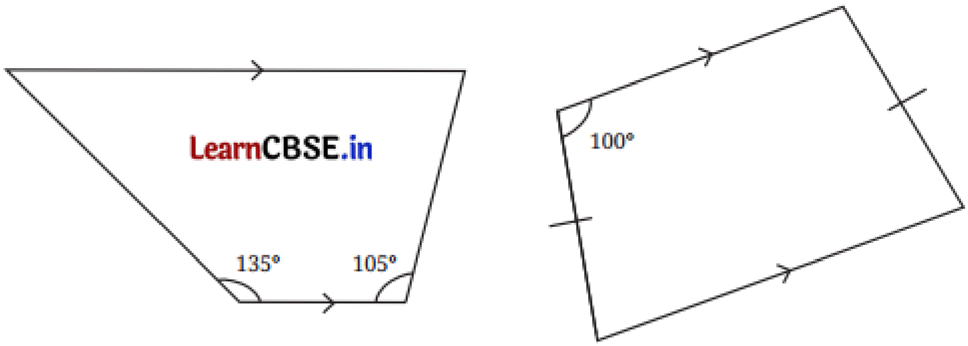
The sides of the quadrilateral ABCD are 4 cm each.
The angles of the quadrilateral ABCD are $\angle A = 60° + 60° = 120°$, $\angle B = 60°$, $\angle C = 60° + 60° = 120°$ and $\angle D = 60°$.

Question 2.
Construct a kite whose diagonals are of lengths 6 cm and 8 cm.
Solution:
Draw a line AB equal to 8 cm.
Take a point P on the line AB.
Draw a perpendicular line to AB passing through P.
Take points C and D on this perpendicular such that PC = PD = 3 cm.
Join AC, CB, BD, and DA.

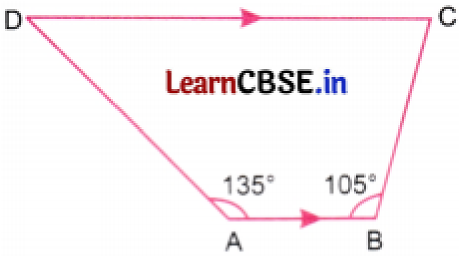


ACBD is the required kite with diagonals of lengths 6 cm and 8 cm.

Question 3.
Find the remaining angles in the following trapeziums.



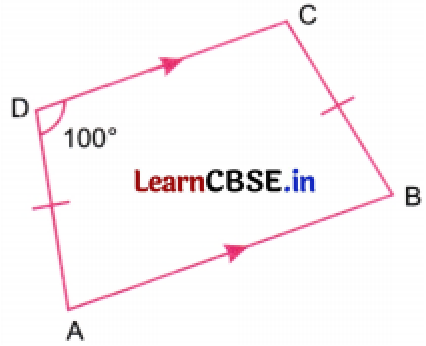
Solution:
(i) Let the given trapezium be ABCD.
Lines AB and DC are Parallel



∴ ∠A + ∠D = 180° and ∠B + ∠C = 180°.
∴ ∠A + ∠D = 180°
⇒ 135° + ∠D = 180°
⇒ ∠D = 180° – 135° = 45°.

∴ ∠B + ∠C = 180°
⇒ 105° + ∠C = 180°
⇒ ∠C = 180° – 105° = 75°.
∴ The remaining angles are 45° and 75°.

(ii) Let the given trapezium be ABCD.
Since AD = BC, ABCD is an isosceles trapezium.



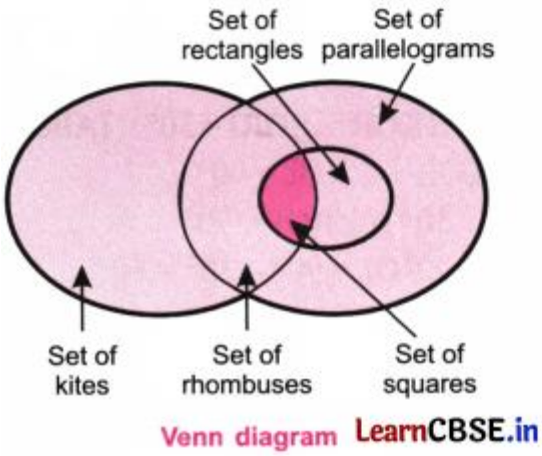
∴ Angles opposite to the equal sides are equal.
∴ ∠C = ∠D = 100°
Lines AB and DC are parallel.
∴ ∠A + ∠D = 180° and ∠B + ∠C = 180°
∴ ∠A + ∠D = 180°
⇒ ∠A + 100° = 180°
⇒ ∠A = 180° – 100°
⇒ ∠A = 80°
∴ ∠B + ∠C = 180°
⇒ ∠B + 100° = 180°
⇒ ∠B = 180° – 100°
⇒ ∠B = 80°
∴ Remaining angles are ∠A = 80°, ∠B = 80° and ∠C = 100°.

Question 4.
Draw a Venn diagram showing the set of parallelograms, kites, rhombuses, rectangles, and squares. Then answer the following questions.
(i) What is the quadrilateral that is both a kite and a parallelogram?
(ii) Can there be a quadrilateral that is both a kite and a rectangle?
(iii) Is every kite a rhombus? If not, what is the correct relationship between these two types of quadrilaterals?
Solution:
We know the following:

- Every rectangle is a parallelogram.
- Every square is a rectangle.

- Every square is a rhombus.
- Every rhombus is a kite.

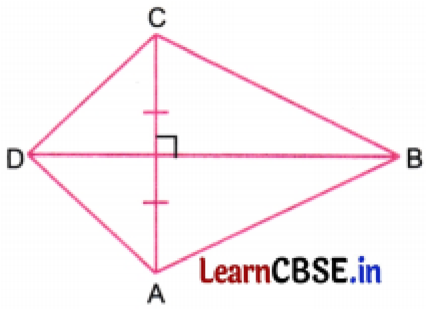
The following Venn diagram shows the sets of parallelograms, kites, rhombuses, rectangles, and squares.



(i) The set of rhombuses is common to both the set of kites and the set of parallelograms.
∴ A rhombus is both a kite and a parallelogram.

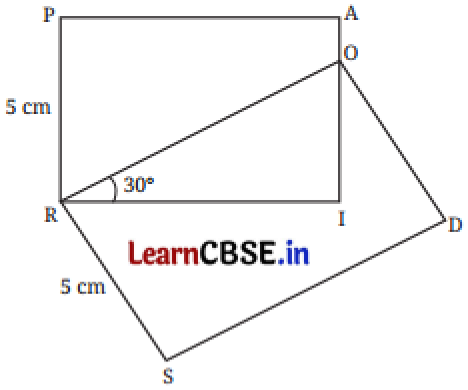
(ii) A kite is not a rectangle, and a rectangle is not a kite.
∴ There can be no quadrilateral that is both a kite and a rectangle.
Also, there is no common portion of the set of kites and the set of rectangles.

(iii) Every kite is not a rhombus.
In the given figure, the kite ABCD is not a rhombus.

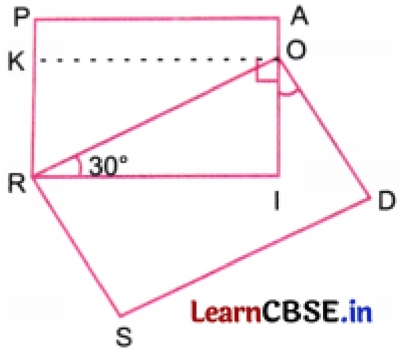


The correct relationship is that every rhombus is a kite.

Question 5.
If PAIR and RODS are two rectangles, find ∠IOD.

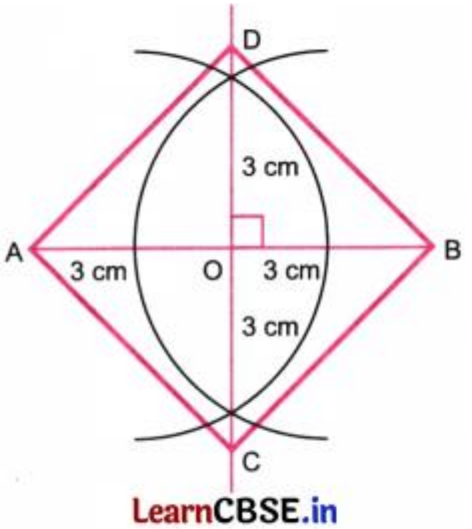


Solution:
From O, draw a line OK parallel to RI.



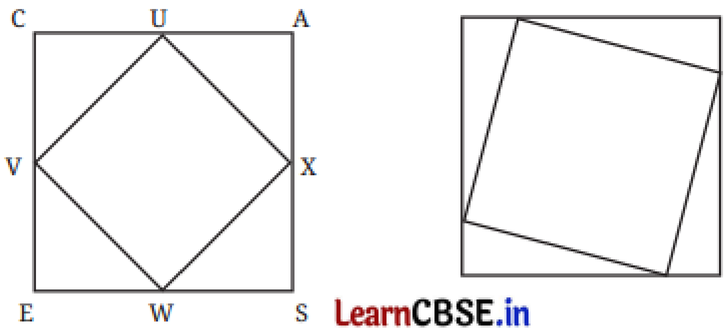
∴ ∠KOR = ∠ORI = 30° (Alternate angles)
∴ ∠KOR + ∠ROI = 90°
⇒ 30° + ∠ROI = 90°
⇒ ∠ROI = 90° – 30°
⇒ ∠ROI = 60°
Also, ∠ROI + ∠IOD = 90°
⇒ 60° + ∠IOD = 90°
⇒ ∠IOD = 90° – 60°
⇒ ∠IOD = 30°.

Question 6.
Construct a square with a diagonal of 6 cm without using a protractor.
Solution:
Draw a line AB equal to 6 cm.



We have 6 ÷ 2 = 3.
With centre at A and B, draw arcs of radius slightly greater than 3 cm, say, 4 cm.
Join the points of intersection of the arcs.
Let this line intersect AB at O.
Take points C and D on the perpendicular line so that OC = OD = 3 cm.
Join AC, CB, BD, and DA.
ACBD is the required square with diagonals equal to 6 cm.

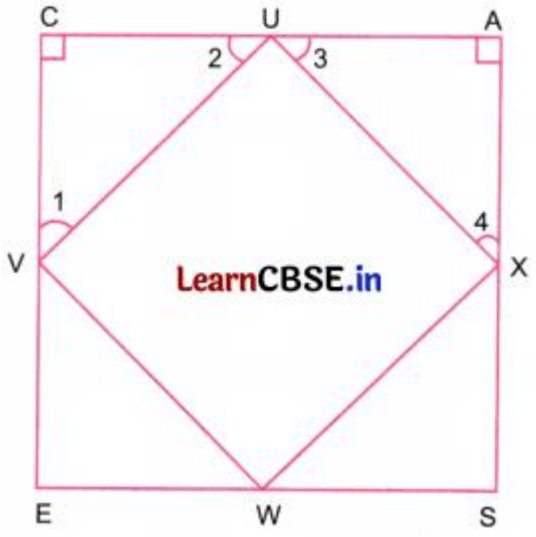
Question 7.
CASE is a square. The points U, V, W, and X are the midpoints of the sides of the square. What type of quadrilateral is UVWX? Find this by using geometric reasoning, as well as by construction and measurement. Find other ways of constructing a square within a square such that the vertices of the inner square lie on the sides of the outer square, as shown in Figure (b).



(a) (b)

Solution:
(a) U, V, W, and X are the midpoints of the sides of the square.
In ΔVCU and ΔUAX,
we have VC = UA, ∠VCU = ∠UAX = 90°, and CU = AX.
∴ By the SAS condition, ΔVCU and ΔUAX are congruent.
∴ VU = UX
Similarly, VU = XW, VU = WV.

∴ Sides of the quadrilateral UVWX are equal.



In ΔVCU, VC = CU

⇒∠1 = ∠2

Also, ∠1 + ∠C + ∠2 = 180°

⇒∠1 + 90° + ∠1 = 180°

⇒2∠1 = 90°

⇒∠1 = 45°

∴ ∠2 is also 45°.

Similarly, ∠3 = ∠4 = 45°

We have ∠2 + ∠VUX + ∠3 = 180°

⇒45° + ∠VUX + 45° = 180°

⇒∠VUX = 180° – 90°

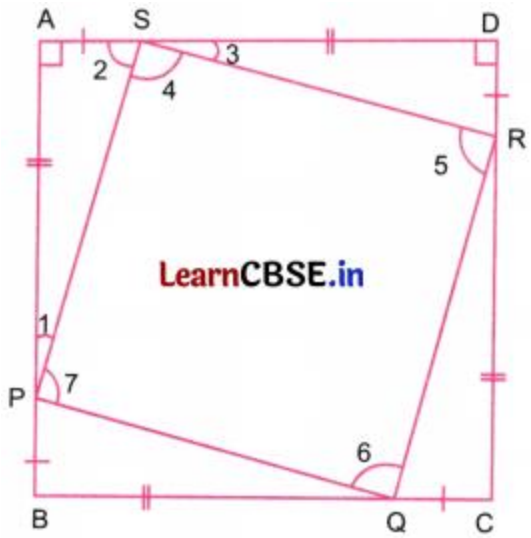
⇒ ∠VUX = 90°

Similarly, ∠VXW = 90°, ∠XWV = 90° and ∠WVU = 90°.

∴ By definition, the quadrilateral UVWX is a square.

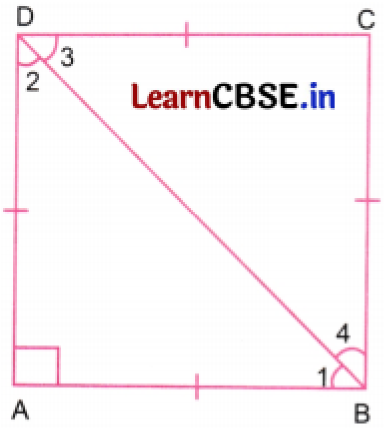
(b) Let ABCD be a square.

Take points P, Q, R, and S such that AS = BP = CQ = DR.



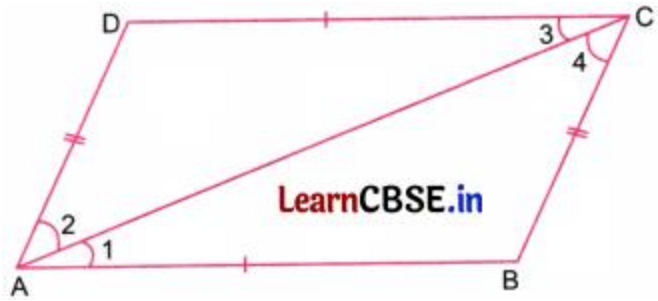
Since the sides of squares are equal,
we have DS = AP = BQ = CR.
In ΔPAS and ΔSDR, we have
PA = SD, ∠PAS = ∠SDR = 90°, and AS = DR.
∴ By the SAS condition, ΔPAS and ΔSDR are congruent.
∴ PS = SR
Similarly, PS = RQ, PS = QP.
∴ Sides of the quadrilateral PQRS are equal.
In ΔPAS, ∠1 + ∠2 + 90° = 180°
⇒∠1 + ∠2 = 90°
⇒∠3 + ∠2 = 90° (∵ ∠1 = ∠3)
Also, ∠2 + ∠4 + ∠3 = 180°
⇒ 90° + ∠4 = 180°
⇒ ∠4 = 180° – 90°
⇒ ∠4 = 90°
∴ Similarly, ∠5 = 90°, ∠6 = 90°, and ∠7 = 90°.
By definition, the quadrilateral PQRS is a square.

Question 8.
If a quadrilateral has four equal sides and one angle of 90°, will it be a square? Find the answer using geometric reasoning as well as by construction and measurement.
Solution:
Let ABCD be a quadrilateral such that AB = BC = CD = DA and ∠DAB = 90°.
Join BD.



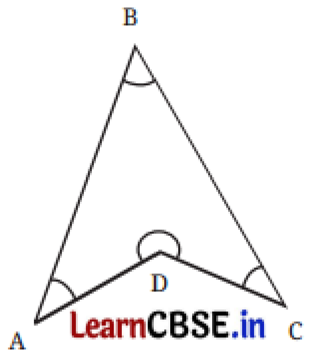
In ΔADB and ΔCDB, we have
AD = CD, AB = CB, and DB is a common side.
∴ ΔADB and ΔCDB are congruent.
∴ ∠C = ∠A = 90°
In ΔDAB, ∠1 = ∠2 (∵ AB = AD)
Also, ∠1 + 90° + ∠2 = 180°
⇒ ∠1 + ∠2 = 90°
⇒ ∠1 = 45° and ∠2 = 45° (∵ ∠1 = ∠2)
In ΔCDB, ∠3 = ∠4 (∵ CD = CB)
Also, ∠3 + 90° + ∠4 = 180°
⇒ ∠3 + ∠4 = 90°
⇒ ∠3 = ∠4 = 45° (∵ ∠3 = ∠4)
∴ ∠ABC = ∠1 + ∠4 = 45° + 45° = 90°
and ∠ADC = ∠2 + ∠3 = 45° + 45° = 90°.
∴ Each angle of the quadrilateral ABCD is 90°.
∴ ABCD is a square.
Also, by measurement, we find AB = BC = CD = DA and ∠A = ∠B = ∠C = ∠D = 90°.

Question 9.
What type of quadrilateral is one in which the opposite sides are equal? Justify your answer.
Hint: Draw a diagonal and check for congruent triangles.
Solution:
Let ABCD be a quadrilateral in which opposite sides are equal.
Join AC.

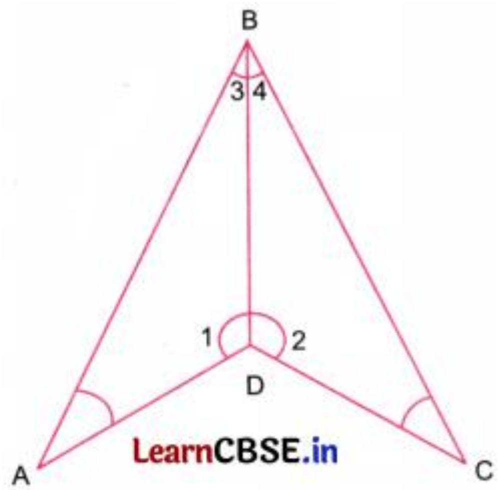


In ΔADC and ΔCBA, we have
AD = CB, DC = BA, and AC is common.
∴ By the SSS condition, AADC and ACBA are congruent.
∴ ∠1 = ∠3 and ∠2 = ∠4
AC is a transversal of lines AB and DC, and alternate angles ∠1 and ∠3 are equal.
∴ Lines AB and DC are parallel.
AC is a transversal of lines AD and BC, and alternate angles ∠2 and ∠4 are equal.
∴ Lines AD and BC are parallel.
∴ By definition, the quadrilateral ABCD is a parallelogram.

Question 10.
Will the sum of the angles in a quadrilateral, such as the following one, also be 360°? Find the answer using geometric reasoning as well as by constructing this figure and measuring.



Solution:
In the given quadrilateral, join BD.
In ΔABD, we have
∠A + ∠3 + ∠1 = 180°
In ΔCBD, we have
∠C + ∠4 + ∠2 = 180°



Adding, we get

$(\angle A + \angle 3 + \angle 1) + (\angle C + \angle 4 + \angle 2) = 180° + 180°$

$\Rightarrow \angle A + (\angle 3 + \angle 4) + \angle C + (\angle 1 + \angle 2) = 360°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

∴ The sum of the angles of the quadrilateral ABCD is 360°.

Also, by using a protractor, we find that the sum of all angles is 360°.

Question 11.

State whether the following statements are true or false. Justify your answers.

(i) A quadrilateral whose diagonals are equal and bisect each other must be a square.

(ii) A quadrilateral having three right angles must be a rectangle.

(iii) A quadrilateral whose diagonals bisect each other must be a parallelogram.

(iv) A quadrilateral whose diagonals are perpendicular to each other must be a rhombus.

(v) A quadrilateral in which the opposite angles are equal must be a parallelogram.

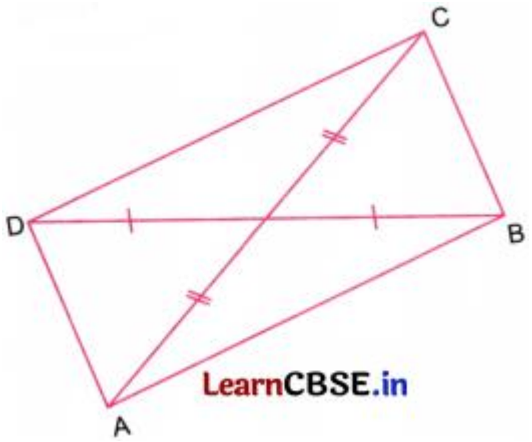
(vi) A quadrilateral in which all the angles are equal is a rectangle.

(vii) Isosceles trapeziums are parallelograms.

Solution:

(i) A quadrilateral whose diagonals are equal and bisect each other need not be a square.

In the figure, diagonals AC and DB are equal and bisect each other. Such a quadrilateral is always a rectangle.

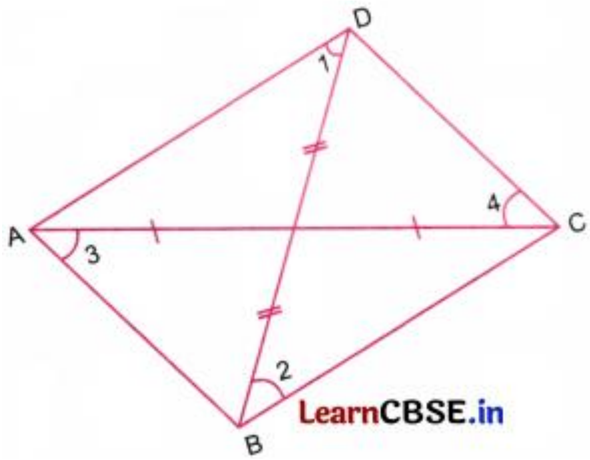


∴ The given statement is false.

(ii) Let ABCD be a quadrilateral having three right angles at A, D, and C.
We have ∠A + ∠B + ∠C + ∠D = 360°.
⇒ 90° + ∠B + 90° + 90° = 360°
⇒ ∠B = 360° – 270°
⇒ ∠B = 90°.
∴ Each angle of ABCD is 90°.
∴ Given quadrilateral is a rectangle.

∴ The given statement is true.

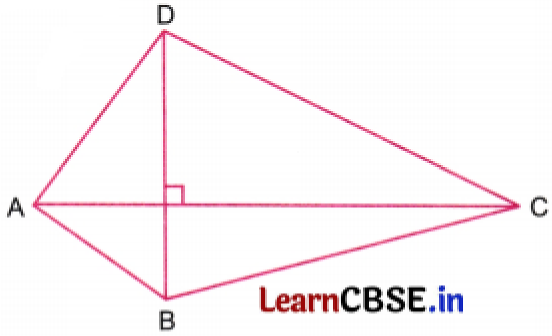
(iii) In the quadrilateral ABCD, the diagonals AC and BD bisect each other.
Here, ΔAOD and ΔCOB are congruent.
∴ ∠1 = ∠2



∴ BC is parallel to AD.

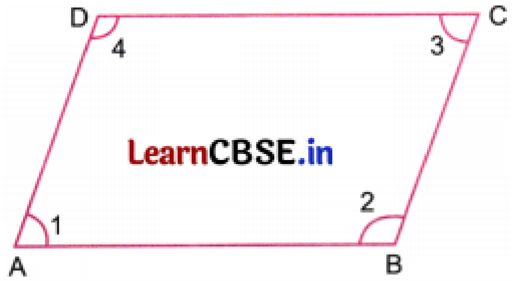
Also, ΔAOB and ΔCOD are congruent.
∴ ∠3 = ∠4
∴ AB is parallel to DC.
Since opposite sides of ABCD are parallel, it must be a parallelogram.
∴ The given statement is true.

(iv) Let ABCD be a quadrilateral whose diagonals AC and BD are perpendicular to each other.



This quadrilateral may not be a rhombus, because the diagonals AC and BC may not bisect each other.
∴ The given statement is false.

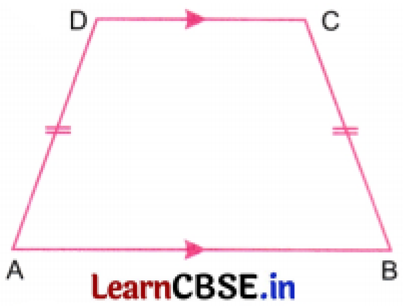
(v) Let ABCD be a quadrilateral in which ∠1 = ∠3 and ∠2 = ∠4.
We have, ∠1 + ∠2 + ∠3 + ∠4 = 360°.
⇒ ∠1 + ∠2 + ∠1 + ∠2 = 360°
⇒ 2(∠1 + ∠2) = 360°
⇒ ∠1 + ∠2 = 180°



AB is a transversal of lines AD and BC, and the sum of internal angles ∠1 and ∠2 on the same side is 180°.
∴ Lines AD and BC are parallel.
Again, ∠1 + ∠2 + ∠3 + ∠4 = 360°
⇒ ∠3 + ∠2 + ∠3 + ∠2 = 360°
⇒ 2(∠2 + ∠3) = 360°
⇒ ∠2 + ∠3 = 180°
BC is a transversal of lines AB and DC, and the sum of internal angles ∠2 and ∠3 on the same sides is 180°.
∴ Lines AB and DC are parallel.

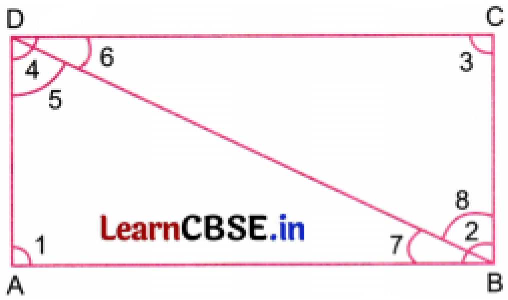
∴ Opposite sides of quadrilateral ABCD are parallel.
∴ ABCD is a parallelogram.
∴ The given statement is true.

(vi) Let ABCD be a quadrilateral, where ∠1, ∠2, ∠3, and ∠4 are all equal.
We have ∠1 + ∠2 + ∠3 + ∠4 = 360°
∴ ∠1 + ∠1 + ∠1 + ∠1 = 360°
⇒ 4∠1 = 360°
⇒ ∠1 = 90°



∴ ∠2 = 90°, ∠3 = 90°, ∠4 = 90°
We have, ∠5 + ∠6 = 90°
and ∠6 + 90° + ∠8 = 180°
⇒ ∠5 + ∠6 = ∠6 + ∠8
⇒ ∠5 = ∠8
Also, ∠7 + 90° + ∠5 = 180°
⇒ ∠7 + ∠5 = 90°
⇒ ∠5 + ∠6 = ∠7 + ∠5
⇒ ∠6 = ∠7
In ΔDAB and ΔBCD, we have ∠5 = ∠8, ∠7 = ∠6, and side BD is common.
∴ By the ASA condition, ΔDAB and ΔBCD are congruent.
∴ DA = BC and AB = CD
∴ Opposite sides of ABCD are equal.
∴ ABCD is a rectangle.
∴ The given statement is true

(vii) An isosceles trapezium ABCD can not be a parallelogram because it has two non-parallel equal lines AD and BC.



∴ The given statement is false.

Practice Time 4.1

Q.1

Two angles of a quadrilateral are $72°$and $98°$. The other two angles are equal. Find them.

Let each of the equal angles be $x°$.

Sum of angles of a quadrilateral = $360°$

$$72° + 98° + x + x = 360°$$
$$170° + 2x = 360°$$
$$2x = 190°$$
$$x = 95°$$

Answer:

$$\boxed{95°, 95°}$$

Q.2

Find $\angle ABC$.

Given:

- $\angle$between the slant line and horizontal line at $O = 70°$
- $\angle A = 90°$
- $BC \parallel OA$

Since $BC \parallel OA$, the angle made by transversal $OC$with $BC$is equal to the angle made by $OC$with OA.

Therefore,

$$\angle BCA = 70^\circ$$

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$
$$90^\circ + \angle ABC + 70^\circ = 180^\circ$$
$$\angle ABC = 20^\circ$$

Answer:

$$\boxed{\angle ABC = 20^\circ}$$

Q.3

In the given trapezium $ABCD$, find x, y, z.

Given:

- $\angle A = 120^\circ$
- Exterior angle at $C = 100^\circ$

Step 1: Find y

$y$and 100°form a linear pair.

$$y + 100^\circ = 180^\circ$$
$$y = 80^\circ$$

Step 2: Find x

Since $AB \parallel DC$,

Interior angles on same side of transversal $BC$are supplementary.

$$x + y = 180^\circ$$
$$x + 80^\circ = 180^\circ$$
$$x = 100^\circ$$

Step 3: Find z

Along transversal AD,

$$120^\circ + z = 180^\circ$$
$$z = 60^\circ$$

Answer:

$$\boxed{x = 100^\circ,\ y = 80^\circ,\ z = 60^\circ}$$

Q.4

Find x.

The $70^\circ$and 80°shown are exterior angles.

Hence interior left angles are:

$$180^\circ - 70^\circ = 110^\circ$$
$$180^\circ - 80^\circ = 100^\circ$$

The top and bottom angles are equal and each is x.

Sum of angles of quadrilateral:

$$110^\circ + 100^\circ + x + x = 360^\circ$$
$$210^\circ + 2x = 360^\circ$$
$$2x = 150^\circ$$
$$x = 75^\circ$$

Answer:

$$\boxed{x = 75^\circ}$$

Q.5 (i) Kite: Find $x$and y

Given:

- Left angle = 110°
- Bottom angle = 60°

In a kite, one pair of opposite angles are equal.

$$y = 110^\circ$$

Sum of angles of a quadrilateral = $360°$

$$x + 110° + 110° + 60° = 360°$$
$$x + 280° = 360°$$
$$x = 80°$$

Answer:

$$\boxed{x = 80°,\ y = 110°}$$

Q.5 (ii) Parallelogram: Find x, y, z

At the intersection of diagonals:

$$100° + x = 180°$$
$$x = 80°$$

Since $AB \parallel DC$, diagonal $DB$is a transversal.

Alternate interior angles are equal:

$$z = 30°$$

In $\triangle BOC$,

$$30° + y + 80° = 180°$$
$$y = 70°$$

Answer:

$$\boxed{x = 80°,\ y = 70°,\ z = 30°}$$

Q.5 (iii) Parallelogram: Find $x$and y

Opposite angles of a parallelogram are equal.

$$6y = 120°$$
$$y = 20$$

Adjacent angles are supplementary.

$$120^\circ + (5x + 10)^\circ = 180^\circ$$
$$5x + 130 = 180$$
$$5x = 50$$
$$x = 10$$

Answer:

$$\boxed{x = 10,\ y = 20}$$

Q.6

Given:

$$PQ \parallel SR,\ PS \parallel QR$$

Therefore $PQRS$is a parallelogram.

At Q,

$$\angle Q = 120^\circ$$

Opposite angles are equal:

$$y = \angle S = 120^\circ$$

Adjacent angles are supplementary:

$$x + 120^\circ = 180^\circ$$
$$x = 60^\circ$$

Opposite angles are equal:

$$z = \angle R = \angle P = 60^\circ$$

Answer:

$$\boxed{x = 60^\circ,\ y = 120^\circ,\ z = 60^\circ}$$

Q.7

In parallelogram $PQRS$:

$$\angle P = \angle R = 110^\circ$$
$$\angle Q = \angle S = 70^\circ$$

Bisector of $\angle P$:

$$\frac{110^\circ}{2} = 55^\circ$$

Bisector of $\angle Q$:

$$\frac{70^\circ}{2} = 35^\circ$$

In $\triangle PMQ$,

$$\angle MPQ = 55^\circ$$
$$\angle MQP = 35^\circ$$
$$\angle PMQ = 180^\circ - (55^\circ + 35^\circ)$$
$$\angle PMQ = 90^\circ$$

Answer:

$$\boxed{\angle PMQ = 90^\circ}$$

Q.8 (i)

Diagonals of a parallelogram bisect each other.

$$LO = OG$$
$$3x - 18 = 15$$
$$3x = 33$$
$$x = 11$$

Answer:

$$\boxed{x = 11}$$

Q.8 (ii)

Diagonals bisect each other.

$$AO = OC$$
$$3x - 8 = x + 10$$
$$2x = 18$$
$$x = 9$$

Answer:

$$\boxed{x = 9}$$

Q.9

Adjacent angles of a parallelogram are supplementary.

$$(3x + 7) + (7x + 3) = 180^\circ$$
$$10x + 10 = 180$$
$$10x = 170$$
$$x = 17$$

Angles:

$$3(17) + 7 = 58^\circ$$
$$7(17) + 3 = 122^\circ$$

Answer:

$$\boxed{58^\circ,\ 122^\circ}$$

(Opposite angles are also $58^\circ$and 122°.)

Q.10 Construction

To draw a parallelogram with sides 3 cm, 4 cm and included angle 45°:

1. Draw $AB = 4$cm.
2. At A, construct $\angle DAB = 45°$.
3. On this ray, mark $AD = 3$cm.
4. Through D, draw a line parallel to AB.
5. Through B, draw a line parallel to AD.
6. Let these lines meet at C.

Then $ABCD$is the required parallelogram.

Practice Time 4.2

Q1. One of the diagonals of a rhombus is equal to one of its sides. Find the angles of the rhombus.

Let side of rhombus = a.

Given, one diagonal = a.

In a rhombus, diagonals bisect each other at right angles.

Let diagonal $AC = a$.

Then,

$$AO = \frac{a}{2}$$

In right $\triangle AOB$,

$$AB = a, AO = \frac{a}{2}$$

Using Pythagoras:

$$OB = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}a}{2}$$

Now,

$$\cos \angle BAO = \frac{AO}{AB} = \frac{\frac{a}{2}}{a} = \frac{1}{2}$$
$$\angle BAO = 60°$$

Diagonal bisects the angle of a rhombus.

$$\angle A = 2 \times 60^\circ = 120^\circ$$
$$\angle B = 180^\circ - 120^\circ = 60^\circ$$

Answer:

$$\boxed{120^\circ,\ 60^\circ,\ 120^\circ,\ 60^\circ}$$

Q2. PQRS is a rhombus. PQ = 10 cm and QS = 16 cm. Find PR.

Diagonals bisect each other at right angles.

$$QM = \frac{16}{2} = 8 \text{ cm}$$
$$PM = \frac{PR}{2}$$

Using Pythagoras in $\triangle PMQ$:

$$10^2 = 8^2 + \left(\frac{PR}{2}\right)^2$$
$$100 = 64 + \left(\frac{PR}{2}\right)^2$$
$$\left(\frac{PR}{2}\right)^2 = 36$$
$$\frac{PR}{2} = 6$$
$$PR = 12 \text{ cm}$$

Answer:

$$\boxed{12 \text{ cm}}$$

Q3. Construct a rhombus whose diagonals are 7 cm and 5 cm.

Construction Steps

1. Draw diagonal $AC = 7\text{cm}$.
2. Mark midpoint $O$of AC.
3. Draw a perpendicular line through O.
4. On this perpendicular, mark

$$OB = OD = 2.5 \text{ cm}$$

(half of 5 cm).

5. Join A, B, C, D.

$$ABCD$$

is the required rhombus.

Q4. In rectangle ABCD, AB = 25 cm and BC = 15 cm. In what ratio does the bisector of $\angle C$divide AB?

Let angle bisector of $\angle C$meet ABat P.

Since $\angle C = 90°$,

$$\angle BCP = 45°$$

Thus $\triangle BCP$is an isosceles right triangle.

$$BP = BC = 15 \text{ cm}$$
$$AP = AB - BP = 25 - 15 = 10 \text{ cm}$$

Ratio:

$$AP:PB = 10:15 = 2:3$$

Answer:

$$\boxed{2:3}$$

Q5. PQRS is a rectangle. The perpendicular ST from S on PR divides $\angle S$in the ratio 2 : 3. Find $\angle TPQ$.

In a rectangle,

$$\angle S = 90^\circ$$

Let parts be $2x$and $3x$.

$$\begin{aligned} &2x + 3x = 90^\circ \\ &5x = 90^\circ \\ &x = 18^\circ \end{aligned}$$

Angles are:

$$36^\circ,\ 54^\circ$$

Since $ST \perp PR$,

the diagonal PRmakes angle

$$90^\circ - 54^\circ = 36^\circ$$

with side PQ.

Answer:

$$\boxed{36^\circ}$$

Q6. Rectangle sides are $(2x+8)$cm and $(2x+1)$cm. Perimeter = 34 cm.

$$\begin{aligned} &2[(2x + 8) + (2x + 1)] = 34 \\ &2(4x + 9) = 34 \\ &8x + 18 = 34 \\ &8x = 16 \\ &x = 2 \end{aligned}$$

Sides:

$$2x + 8 = 12 \text{ cm}$$

$$2x + 1 = 5 \text{ cm}$$

Diagonal:

$$d = \sqrt{12^2 + 5^2}$$
$$d = \sqrt{169}$$
$$d = 13 \text{ cm}$$

Answer:

Sides:

$$\boxed{12 \text{ cm and } 5 \text{ cm}}$$

Diagonals:

$$\boxed{13 \text{ cm each}}$$

Q7. Diagonals are equal (7 cm) and bisect each other at 45°.

A quadrilateral whose diagonals are:

- equal
- bisect each other

is a rectangle.

A rectangle has diagonals intersecting at 45°only when it becomes a square.

Answer:

$$\boxed{\text{Square}}$$

Q8. Construct a square with diagonal 4 cm.

Construction

1. Draw diagonal $AC = 4$cm.
2. Find midpoint $O$of AC.

3. Draw perpendicular bisector of AC through O.
4. Mark points B and D on it such that

$$OB = OD = 2 \text{ cm}$$

5. Join AB, BC, CD, DA.

Required square $ABCD$ is obtained.

Q9. True/False

(i) All rectangles are parallelograms.

True

(ii) Diagonals of rectangle intersect at right angles.

False

(iii) Diagonals of rhombus intersect at right angles.

True

(iv) In a square, diagonals intersect at right angles but not equal.

False (they are equal as well)

EXAM TIME

A. Multiple Choice Questions

1. The angle measurements of a quadrilateral are 35°, 49°, 67°. The measure of the fourth angle is

Sum of angles of a quadrilateral = 360°

Fourth angle
= 360° − (35° + 49° + 67°)

= 360° − 151°

= **209°**

Answer: (b) 209°

---

2. ABCD is a quadrilateral, in which AB = 5 cm, CD = 8 cm and the sum of angles ∠A and ∠D is 180°. What is the name of this quadrilateral?

Given:

$\angle A + \angle D = 180°$

This suggests one pair of sides may be parallel, but only one pair of supplementary adjacent angles is not enough to conclude a definite type.

Also,

AB = 5 cm and CD = 8 cm

So opposite sides are not equal.

The given information is insufficient to identify the quadrilateral uniquely.

Answer: (d) Cannot be determined

---

3. The angles of a quadrilateral ABCD taken in order are in the ratio 3 : 7 : 6 : 4. Then, the ABCD is a

Let the angles be:

3x, 7x, 6x, 4x

Sum of angles of a quadrilateral = 360°

$3x + 7x + 6x + 4x = 360°$

$20x = 360°$

$x = 18°$

Therefore,

$\angle A = 54°$

$\angle B = 126°$

$\angle C = 108°$

$\angle D = 72°$

Now,

$\angle A + \angle B = 180°$

and

$\angle C + \angle D = 180°$

Hence one pair of opposite sides is parallel.

Therefore the quadrilateral is a **trapezium**.

Answer: (c) Trapezium

---

4. The diagonals of a kite

Properties of a kite:

- One diagonal bisects the other.
- Diagonals are perpendicular to each other.

Therefore,

Answer: (b) are perpendicular to each other

5. The diagonals of a rectangle are

(a) perpendicular to each other
(b) equal and bisect each other
(c) unequal
(d) bisect at right angles

A rectangle has diagonals that are **equal in length and bisect each other**.

Answer: (b) equal and bisect each other

---

6. One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.

Let side = diagonal.

In a rhombus all sides are equal.

A diagonal divides the rhombus into two equilateral triangles.

Therefore each angle of the equilateral triangle = 60°.

Hence angles of rhombus are:

60°, 120°, 60°, 120°

Answer: 60° and 120°

---

7. Vanita folds a rectangular sheet. The folded sheet is shown below. What is the length of the sheet?

Top side is divided into:

5 cm + 3 cm = 8 cm

Since folding creates equal distances from the fold line, the rectangle formed has dimensions 8 cm × 8 cm.

Hence length = **8 cm**

Answer: (a) 8 cm

---

8. In a parallelogram ABCD, ∠A and ∠B are in the ratio 2 : 3. Find ∠A.

Adjacent angles of a parallelogram are supplementary.

Let

$\angle A = 2x$

$\angle B = 3x$

$2x + 3x = 180°$

$5x = 180°$

$x = 36°$

$\angle A = 2 \times 36°$

= **72°**

Answer: (b) 72°

---

9. If the diagonals of a parallelogram bisect each other at right angles, then it will be a

A rhombus has diagonals that:

- bisect each other
- intersect at right angles

Answer: (a) Rhombus

---

10. Which one has all the properties of a kite and a parallelogram?

A rhombus:

- has all sides equal
- has two pairs of adjacent equal sides (kite property)
- opposite sides parallel (parallelogram property)

Answer: (b) Rhombus

---

11. The diagonals of a rectangle are 2x + 1 and 3x – 1. Find x.

In a rectangle, diagonals are equal.

$2x + 1 = 3x - 1$

$3x - 2x = 1 + 1$

$x = 2$

Answer: (b) 2

B. Fill in the Blanks

1. ________ is a regular quadrilateral.

A regular quadrilateral has all sides equal and all angles equal.

Square

---

2. If the diagonals of a quadrilateral are equal and bisect each other at 90°, it is a ________.

A quadrilateral whose diagonals are equal, bisect each other and meet at right angles is a

Square

---

3. If PQRS is a parallelogram then ∠Q − ∠S is equal to ______ degree.

Opposite angles of a parallelogram are equal.

$\angle Q = \angle S$

$\angle Q - \angle S = 0°$

0°

---

4. The adjacent angles of a parallelogram are ________.

Adjacent angles of a parallelogram add up to 180°.

Supplementary

C. True / False

1. Kite is a parallelogram, in which each pair of opposite sides are parallel.

A kite generally does not have opposite sides parallel.

False

---

2. PQRS is a trapezium, in which PQ ∥ RS and ∠P = 130°, ∠Q = 110° then ∠R is equal to 70°.

Since PQ ∥ RS,

$\angle Q + \angle R = 180°$

$110° + \angle R = 180°$

$\angle R = 70°$

True

---

3. The sum of adjacent angles of a parallelogram is 180°.

Adjacent angles are supplementary.

True

---

4. If the adjacent angles of a parallelogram are equal then the parallelogram is a rhombus.

Equal adjacent angles imply each is 90°.

That gives a rectangle, not necessarily a rhombus.

False

---

5. The diagonals of a rhombus bisect each other at right angles.

This is a standard property of a rhombus.

True

D. Match the Columns

Column I	Reason	Column II
(a) If AB = AD and BC = CD	Two pairs of adjacent sides are equal	**(ii) Kite**
(b) If AB ∥ CD and AD = BC	One pair of opposite sides parallel and non-parallel sides equal	**(iii) Isosceles trapezium**
(c) If AB ∥ CD and BC ∥ AD	Both pairs of opposite sides parallel	**(iv) Parallelogram**
(d) If AC = BD and ABCD is a parallelogram	A parallelogram with equal diagonals is a rectangle	**(v) Rectangle**
(e) A regular quadrilateral	All sides equal and all angles equal	**(i) Square**

Answer:

$$\boxed{(a) \rightarrow (ii)}$$
$$\boxed{(b) \rightarrow (iii)}$$
$$\boxed{(c) \rightarrow (iv)}$$
$$\boxed{(d) \rightarrow (v)}$$
$$\boxed{(e) \rightarrow (i)}$$

E. Very Short Answer Type Questions

1. If four angles of a quadrilateral are in the ratio 3 : 8 : 10 : 3, then find all its angles.

Solution

Let the angles be

3x, 8x, 10x and 3x

Sum of angles of a quadrilateral = 360°

$3x + 8x + 10x + 3x = 360°$

$24x = 360°$

$x = 15°$

Therefore,

$\angle 1 = 3 \times 15° = 45°$

$\angle 2 = 8 \times 15° = 120°$

$\angle 3 = 10 \times 15° = 150°$

$\angle 4 = 3 \times 15° = 45°$

Answer:

45°, 120°, 150°, 45°

---

2. In the following figure, find the value of x + y + z + w.

Given interior angles of quadrilateral:

110°, 100°, 60° and z.

Sum of angles of a quadrilateral = 360°

$z = 360° - (110° + 100° + 60°)$

$z = 90°$

Now exterior angles:

$x + 100° = 180°$

$x = 80°$

$y + 60° = 180°$

$y = 120°$

$w + 110° = 180°$

$w = 70°$

Therefore,

x + y + z + w

= 80° + 120° + 90° + 70°

= **360°**

Answer:

360°

---

3. Find all angles of the given trapezium.

Given:

- AD = BC (marked equal)
- ∠A = 70°

Hence the trapezium is **isosceles**.

In an isosceles trapezium,

∠A = ∠B

Therefore,

∠B = 70°

Also,

∠A + ∠D = 180°

70° + ∠D = 180°

∠D = 110°

Similarly,

∠B + ∠C = 180°

70° + ∠C = 180°

∠C = 110°

Answer:

∠A = 70°, ∠B = 70°, ∠C = 110°, ∠D = 110°

---

4. Find the value of x in the trapezium ABCD given below.

Given:

∠B = $(x + 20)°$

∠C = $(x - 30)°$

AB ∥ DC

Consecutive interior angles are supplementary.

$(x + 20)° + (x - 30)° = 180°$

$2x - 10 = 180$

$2x = 190$

$x = 95°$

Answer:

$x = 95°$

---

5. Find the value of x in the given parallelogram.

Given:

∠Q = 100°

∠P = x

In a parallelogram, adjacent angles are supplementary.

$x + 100° = 180°$

$x = 80°$

Answer:

$x = 80°$

F. Short Answer Type Questions

1. In a trapezium ABCD, MC and MD are bisectors of ∠C and ∠D respectively. Find ∠ABC and ∠BAD.

Given:

- MC bisects ∠C
- MD bisects ∠D

- ∠MDC = 30°
- ∠MCD = 40°
- AB ∥ DC

Step 1: Find ∠D

Since MD bisects ∠D,

∠D = 2 × 30°

= **60°**

Step 2: Find ∠C

Since MC bisects ∠C,

∠C = 2 × 40°

= **80°**

Step 3: Find ∠ABC

AB ∥ DC

Interior angles on the same side are supplementary.

∠B + ∠C = 180°

∠B + 80° = 180°

∠B = **100°**

Step 4: Find ∠BAD

∠A + ∠D = 180°

∠A + 60° = 180°

∠A = **120°**

Answer:

∠ABC = 100°
∠BAD = 120°

---

2. PQRS is a trapezium such that PQ ∥ RS, ∠P : ∠S = 2 : 1, ∠Q : ∠R = 7 : 5. Find the angles of the trapezium.

Step 1

Since PQ ∥ RS,

$\angle P + \angle S = 180°$

Let

$\angle P = 2x$

$\angle S = x$

$3x = 180°$

$x = 60°$

Therefore,

$\angle P = 120°$

$\angle S = 60°$

Step 2

Let

$\angle Q = 7y$

$\angle R = 5y$

Since PQ ∥ RS,

$\angle Q + \angle R = 180°$

$12y = 180°$

$y = 15°$

Hence,

$\angle Q = 105°$

$\angle R = 75°$

Answer:

$\angle P = 120°$
$\angle Q = 105°$
$\angle R = 75°$
$\angle S = 60°$

---

3. In a trapezium FARE, EP and RP are bisectors of ∠E and ∠R respectively. Find ∠FAR and ∠EFA.

Given:

EP bisects ∠E

Angle shown = 25°

Therefore

∠E = 2 × 25°

= 50°

RP bisects ∠R

Angle shown = 30°

Therefore

∠R = 2 × 30°

= 60°

Since ER ∥ FA,

Find ∠F

∠E + ∠F = 180°

50° + ∠F = 180°

∠F = 130°

Find ∠A

∠R + ∠A = 180°

60° + ∠A = 180°

∠A = 120°

Answer:

∠FAR = 120°
∠EFA = 130°

---

4. The adjacent angles of a parallelogram are (3x − 4)° and (2x − 1)°. Find all angles of the parallelogram.

Step 1

Adjacent angles of a parallelogram are supplementary.

$(3x - 4) + (2x - 1) = 180$

$5x - 5 = 180$

$5x = 185$

$x = 37$

Step 2

Angles:

$3x - 4$

$= 3(37) - 4$

$= 107°$

$2x - 1$

$= 74 - 1$

$= 73°$

Opposite angles are equal.

Answer:

∠A = 107°
∠B = 73°
∠C = 107°
∠D = 73°

---

5. In a parallelogram PQRS, the bisectors of ∠P and ∠Q meet at O. Find ∠POQ.

Property:

Adjacent angles of a parallelogram are supplementary.

$\angle P + \angle Q = 180°$

The bisectors divide them into half.

Therefore angle between the bisectors

$= \frac{1}{2}(\angle P + \angle Q)$

$= \frac{1}{2}(180°)$

= 90°

Answer:

∠POQ = 90°

---

6. In the given figure, ABCD and BDCE are parallelograms with common base DC. If BC ⊥ BD, then find ∠BEC.

Step 1

In parallelogram ABCD,

AB || DC

AD || BC

Given $\angle A = 30°$

Hence angle between BC and DC = 30°

Step 2

Given $BC \perp BD$

Therefore

$\angle CBD = 90°$

Step 3

In triangle BCD

$\angle BCD = 30°$

$\angle CBD = 90°$

∠BDC = 180° − (90° + 30°)

= 60°

Step 4

In parallelogram BDCE

BE ∥ DC

EC ∥ BD

Therefore

∠BEC = angle between DC and BD

= ∠BDC

= **60°**

Answer:

∠BEC = 60°

6. One of the diagonals of a rhombus and its sides are equal. Find the angles of the rhombus.

Given:

- ABCD is a rhombus.
- One diagonal is equal to its side.

Let:

- Side of rhombus = a
- Diagonal $AC = a$

Step 1: Consider triangle ABC

In a rhombus,

$$AB = BC = a$$

and given

$$AC = a$$

Therefore,

$$AB = BC = AC$$

Hence, $\triangle ABC$is an **equilateral triangle**.

So,

$$\angle ABC = 60°$$

Step 2: Use the property of a rhombus

Diagonal ACbisects angle A.

In equilateral triangle ABC,

$$\angle BAC = 60°$$

Since ACbisects angle A,

$$\angle A = 2 \times 60°$$
$$\angle A = 120°$$

Step 3: Find the remaining angles

Opposite angles of a rhombus are equal.

$$\angle A = \angle C = 120°$$

Adjacent angles are supplementary.

$$\angle B = \angle D = 180° - 120°$$
$$\angle B = \angle D = 60°$$

Answer:

$$\boxed{\angle A = \angle C = 120°}$$
$$\boxed{\angle B = \angle D = 60°}$$

7. In a parallelogram WISH, find ∠SWH, ∠OSH and ∠SHO.

Given:

- Diagonals WS and IH intersect at O.
- $\angle WOH = 110°$
- At vertex I, diagonal IH divides angle I into 35° and 25°.

Therefore,

$\angle I = 35° + 25° = 60°$

Since opposite angles of a parallelogram are equal,

$\angle H = 60°$

Adjacent angles are supplementary,

$\angle W = \angle S = 180° - 60° = 120°$

Step 1: Find ∠SWH

In triangle WOH,

$\angle WHO = 25°$ (alternate interior angles because WH ∥ IS)

$\angle WOH = 110°$

Therefore,

$\angle OWH = 180° - (110° + 25°)$

$= 45°$

Since O lies on WS,

$\angle SWH = \angle OWH$

$= \mathbf{45°}$

Step 2: Find ∠OSH

Since SH ∥ WI,

Angle between WS and SH equals angle between WS and WI.

At I,

$\angle WIO = 35°$

Therefore,

∠OSH = **35°**

Step 3: Find ∠SHO

In ΔOSH,

∠SOH = 110°

∠OSH = 35°

Therefore,

∠SHO = 180° − (110° + 35°)

= **35°**

Answer:

∠SWH = 45°
∠OSH = 35°
∠SHO = 35°

---

8. In the rectangle LEAP, find ∠EAL, ∠LAP and ∠LOP.

Given:

Rectangle LEAP

Diagonals intersect at O.

One angle at O = 60°

Step 1: Find ∠LOP

Adjacent angles at intersection are supplementary.

∠LOP = 180° − 60°

= **120°**

Step 2: Find ∠EAL

In a rectangle, diagonals are equal and bisect each other.

Thus ΔAOL is isosceles.

Vertex angle

= 60°

Base angles

= (180° − 60°)/2

= 60°

Hence

∠EAL = **60°**

Step 3: Find ∠LAP

Rectangle angle at A = 90°

∠LAP = 90° − 60°

= **30°**

Answer:

∠EAL = 60°
∠LAP = 30°
∠LOP = 120°

---

9. If RENT is a rectangle. Its diagonals meet at O. Find the value of x if OR = (5x + 4) and OT = (3x + 12).

Property

Diagonals of a rectangle are equal and bisect each other.

Therefore,

OR = OT

5x + 4 = 3x + 12

2x = 8

x = **4**

Answer:

x = 4

---

10. ABCD is a rectangle with ∠BAC = 68°. Determine the value of ∠DBC.

Property

AB ∥ DC

Rectangle angle B = 90°

Diagonal AC makes 68° with AB.

Therefore diagonal BD makes 68° with DC.

In triangle DBC,

∠BDC = 68°

∠DCB = 90°

Hence,

∠DBC = 180° − (68° + 90°)

= 22°

Answer:

∠DBC = 22°

---

11. Quadrilateral EFGH is a rectangle, in which I is the point of intersection of diagonals. Find x if IF = (8x + 4) and EG = (24x − 8).

Property

In a rectangle,

- Diagonals are equal.
- They bisect each other.

Therefore,

IF = ½ EG

8x + 4 = (24x − 8)/2

16x + 8 = 24x − 8

16 = 8x

x = **2**

Answer:

x = 2

---

12. Construct a square whose length of diagonal is 6 cm.

Construction Steps

1. Draw diagonal AC = 6 cm.
2. Find midpoint O of AC.
3. Draw a perpendicular bisector through O.
4. With O as centre and radius OA (= 3 cm), cut the perpendicular at points B and D.
5. Join AB, BC, CD and DA.

ABCD is the required square.

Verification

- AC = BD = 6 cm
- Diagonals bisect each other at 90°
- All sides are equal

Hence ABCD is a square.

Construction completed.

G. Long Answer Type Questions

1. In the following figure, $AB \parallel DC$ and $AD = BC$. Find the value of x.

Given:

- $AB \parallel DC$
- $AD = BC = 10$cm
- $DC = 20$cm
- $\angle B = 60°$

Since $AD = BC$, the trapezium is an isosceles trapezium.

Drop perpendiculars from D and C to AB.

In right triangle near B,

$$BE = BC\cos\ 60^\circ$$
$$BE = 10 \times \frac{1}{2}$$
$$BE = 5\ \text{cm}$$

Similarly,

$$AF = 5\ \text{cm}$$

Therefore,

$$AB = AF + DC + BE$$
$$AB = 5 + 20 + 5$$
$$AB = 30\ \text{cm}$$

Answer:

$$\boxed{x = 30\ \text{cm}}$$

---

2. In the following figure of a ship, $ABDH$ and $CEFG$ are two parallelograms. Find the value of x.

Given:

- $ABDH$ and $CEFG$ are parallelograms.
- $\angle B = 130^\circ$
- $\angle F = 30^\circ$

Step 1

In parallelogram $ABDH$,

Adjacent angles are supplementary.

$$\angle H = 180^\circ - 130^\circ$$
$$\angle H = 50^\circ$$

Hence diagonal HO makes 50° with the horizontal.

Step 2

In parallelogram $CEFG$,

Side $EG \parallel CF$.

Given

$$\angle F = 30^\circ$$

Therefore GO makes 30° with the horizontal.

Step 3

The angle at O between OH and OG is

$$x = 50^\circ + 30^\circ$$
$$x = 80^\circ$$

Answer:

$$\boxed{x = 80^\circ}$$

---

3. If two adjacent angles of a parallelogram are in the ratio $3:7$, find all the angles of the parallelogram.

Solution

Adjacent angles of a parallelogram are supplementary.

Let them be

$$3x \text{ and } 7x$$

Then,

$$3x + 7x = 180^\circ$$
$$10x = 180^\circ$$
$$x = 18^\circ$$

Thus,

$$3x = 54^\circ$$

$$7x = 126°$$

Opposite angles are equal.

Therefore angles are

$$54°, 126°, 54°, 126°$$

Answer:

$$\boxed{54°, 126°, 54°, 126°}$$

---

4. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle is $45°$. Find the measure of the obtuse angle.

Solution

The two altitudes are perpendicular to the two adjacent sides.

The angle between two perpendiculars is equal to the acute angle between the original sides.

Hence acute angle of the parallelogram

$$= 45°$$

Obtuse angle

$$\begin{aligned} &= 180° - 45° \\ &= 135° \end{aligned}$$

Answer:

$$\boxed{135°}$$

---

5. Two sticks each of length 7 cm are crossing each other such that they bisect each other at right angles. What shape is formed by joining their end points? Give reason.

Solution

When the diagonals of a quadrilateral:

- Bisect each other
- Are perpendicular
- Are equal in length

the quadrilateral formed is a square.

Since both sticks are 7 cm long and bisect each other at right angles,

the figure obtained is a square.

Answer:

$$\boxed{\text{Square}}$$

---

6. RISE is a rectangle and its diagonals meet at O. If $RO = (3x + 15)$and $IO = (5x + 7)$, find the value of x.

Solution

In a rectangle,

- Diagonals are equal.
- Diagonals bisect each other.

Therefore,

$$\begin{aligned} &RO = IO \\ &3x + 15 = 5x + 7 \\ &15 - 7 = 5x - 3x \\ &8 = 2x \\ &x = 4 \end{aligned}$$

Answer:

$$\boxed{x = 4}$$

---

7. A photo frame is in the shape of a quadrilateral with one diagonal longer than the other. Is it a rectangle? Why or why not?

Solution

A rectangle has the property that its diagonals are equal.

Given that one diagonal is longer than the other.

Therefore the diagonals are not equal.

Hence the quadrilateral cannot be a rectangle.

Answer:

No, it is not a rectangle because the diagonals of a rectangle are always equal, whereas here one diagonal is longer than the other.

Competency-Based Questions

A. Assertion–Reason Questions

1.

Assertion (A): A kite is not a quadrilateral.
Reason (R): A quadrilateral has 2 diagonals.

- Assertion is **false** because a kite is a quadrilateral.
- Reason is **true** because every quadrilateral has two diagonals.

Answer:

(d) A is false but R is true.

2.

Assertion (A): In a trapezium, the non-parallel sides are equal in length always.

Reason (R): Trapeziums have one pair of opposite sides that are parallel.

- Assertion is **false**. Only an isosceles trapezium has equal non-parallel sides.
- Reason is **true**.

Answer:

(d) A is false but R is true.

3.

Assertion (A): A square is both a rectangle and a rhombus.

Reason (R): A square has all sides equal and its opposite sides are parallel.

- Assertion is **true**.
- Reason is **true**.
- Reason correctly explains why a square satisfies the properties of both a rectangle and a rhombus.

Answer:

(a) Both A and R are true and R is the correct explanation of A.

B. Case Study Based Questions

1. Playground in the form of rectangle ABCD

Given:

- ABCD is a rectangle.
- BC = EC.

Since opposite sides of a rectangle are perpendicular,

$$\angle BCE = 90^\circ$$

Also,

$$BC = EC$$

Therefore ΔBCE is an isosceles right triangle.

Hence,

$$\angle CBE = \angle BEC = 45^\circ$$

(i) Find x

Angle x is the exterior angle at E.

$$x = 180^\circ - 45^\circ$$
$$x = 135^\circ$$

Answer:

$$\boxed{x = 135^\circ}$$

(ii) Find $2x + 3y$

$$y = 45^\circ$$
$$2x + 3y = 2(135) + 3(45)$$
$$= 270 + 135$$
$$= 405^\circ$$

Answer:

$$\boxed{405^\circ}$$

(iii) What kind of values do you depict from sports person?

- Discipline
- Team spirit
- Hard work
- Honesty
- Cooperation
- Respect for rules
- Determination

Answer:

Sports persons depict **discipline, teamwork, honesty and perseverance**.

---

2. Rectangle MORE

(i) Is RE = OM?

In a rectangle, opposite sides are equal.

Answer:

Yes, RE = OM.

(ii) Is ∠MYO = ∠RXE?

Both are right angles (90°).

Answer:

Yes, ∠MYO = ∠RXE = 90°.

(iii) Is ∠MOY = ∠REX?

Since RE ∥ OM and OE is a transversal,

corresponding angles are equal.

Answer:

Yes, ∠MOY = ∠REX.

(iv) Is ΔMYO ≅ ΔRXE?

We have:

- ∠MYO = ∠RXE
- ∠MOY = ∠REX
- OM = RE

Therefore by **AAS congruence**,

$$\triangle MYO \cong \triangle RXE$$

Answer:

Yes, ΔMYO ≅ ΔRXE.

(v) Is MY = RX?

Corresponding parts of congruent triangles are equal.

Answer:

Yes, MY = RX.

C. Maths Booster – Crossword Answers

Across

1. A quadrilateral with one pair of parallel sides.
   Answer: TRAPEZIUM
2. A simple closed curve made up of only line segments.
   Answer: POLYGON
3. A quadrilateral which has exactly two distinct consecutive pairs of sides of equal length.
   Answer: KITE
4. A line segment connecting two non-consecutive vertices of a polygon.
   Answer: DIAGONAL
5. The diagonals of a rhombus are ______ bisectors of one another.
   Answer: PERPENDICULAR
6. The ______ sides of a rectangle are of equal length.
   Answer: OPPOSITE
7. The sum of measure of the three angles of a ______ is 180°.
   Answer: TRIANGLE

Down

8. The ______ angles of a parallelogram are supplementary.
   Answer: ADJACENT
9. A ______ is a quadrilateral whose pair of opposite sides are parallel.
   Answer: PARALLELOGRAM
10. The diagonals of a rectangle are of ______ length.
    Answer: EQUAL
11. The diagonals of a parallelogram ______ each other.
    Answer: BISECT

12. A quadrilateral having all the properties of a parallelogram and also that of a kite.
Answer: RHOMBUS

Chapter 5 :Number Play

Intext Questions

Question 1

(i) Can I write every natural number as a sum of consecutive numbers?

Answer: No.

Some natural numbers can be written as a sum of consecutive numbers.

Examples:

$$5 = 2 + 3$$
$$9 = 4 + 5$$
$$15 = 7 + 8$$

But some numbers cannot be written as a sum of two or more consecutive natural numbers.

Examples:

$$1$$
$$2$$
$$4$$
$$8$$

Hence, **every natural number cannot be written as a sum of consecutive numbers.**

(ii) Which numbers can I write as the sum of consecutive numbers in more than one way?

Let us check:

$$9 = 4 + 5$$

Also,

$$9 = 2 + 3 + 4$$

So, 9 can be written in two ways.

$$15 = 7 + 8$$
$$15 = 4 + 5 + 6$$
$$15 = 1 + 2 + 3 + 4 + 5$$

So, 15 can be written in three ways.

$$21 = 10 + 11$$
$$21 = 6 + 7 + 8$$
$$21 = 1 + 2 + 3 + 4 + 5 + 6$$

So, 21 can also be written in more than one way.

Answer:

Numbers such as

$$9,\ 15,\ 21,\ 25,\ 27, \ldots$$

can be written as the sum of consecutive numbers in more than one way.

(iii) All odd numbers can be written as a sum of two consecutive numbers. Can all even numbers also be written as a sum of two consecutive numbers?

For two consecutive numbers:

$$n + (n + 1) = 2n + 1$$

The result is always odd.

Examples:

$$3 + 4 = 7$$
$$8 + 9 = 17$$
$$20 + 21 = 41$$

All results are odd.

Therefore, an even number cannot be written as the sum of **two consecutive natural numbers**.

Example:

There are no consecutive natural numbers whose sum is

$$10$$

or

$$18$$

or

$$24.$$

Answer: No, even numbers cannot be written as the sum of two consecutive natural numbers.

(iv) Can I write 0 as a sum of consecutive numbers? Maybe I should use negative numbers.

Yes.

Using consecutive integers:

$$-1 + 0 + 1 = 0$$

Also,

$$-2 - 1 + 0 + 1 + 2 = 0$$

Therefore,

$$0$$

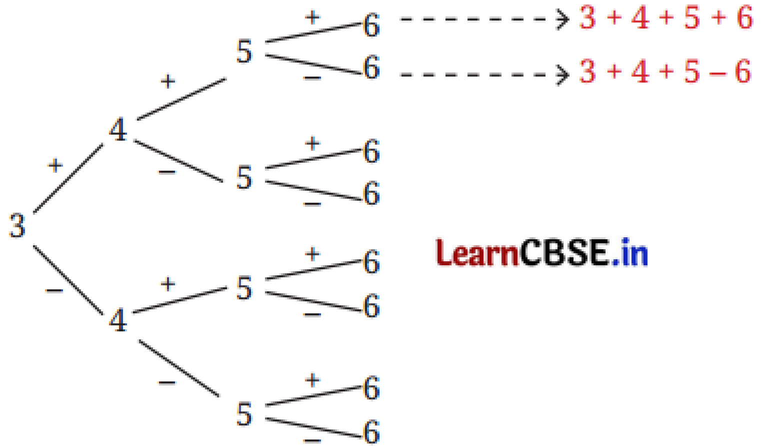
can be written as a sum of consecutive integers when negative numbers are allowed.

Answer: Yes. For example,

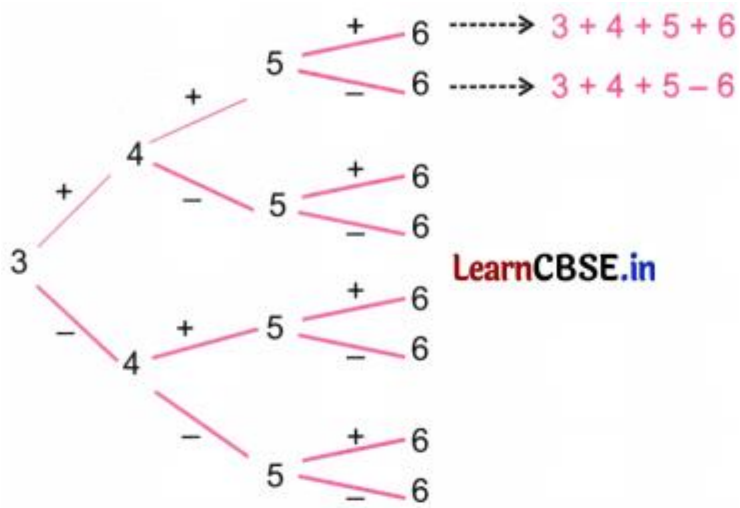
$$-1 + 0 + 1 = 0$$

Question 2.
Take any 4 consecutive numbers. For example, 3, 4, 5, and 6. Place '+' and signs in between the numbers. How many different possibilities exist? Write all of them.

3 + 4 – 5 + 6
3 – 4 – 5 – 6

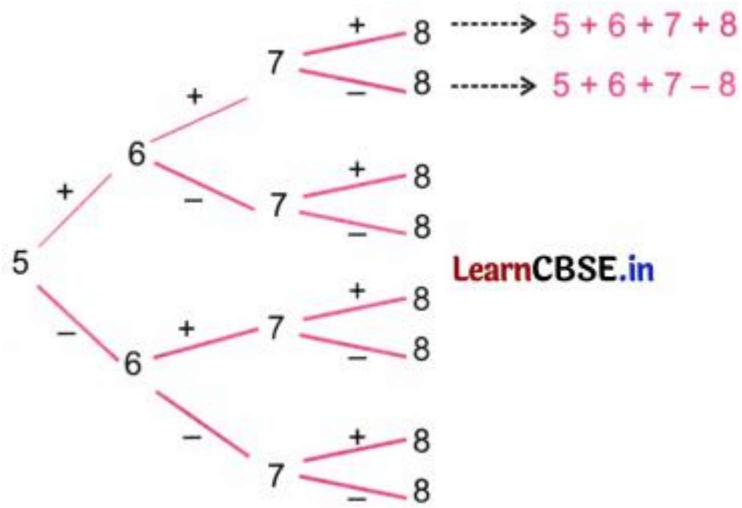


Evaluate each expression and write the result next to it. Do you notice anything interesting?
Solution:
When four consecutive numbers are used with all possible combinations of '+' and signs, the results will always be even. This is because, regardless of the sign placement, the sum will always involve adding and subtracting an equal number of consecutive numbers. Specifically, the sum of four consecutive numbers is always even, and the differences between them also result in even numbers when combined with plus and minus signs.
Example with 3, 4, 5, 6:



3 + 4 + 5 + 6 = 18
3 + 4 + 5 – 6 = 6
3 + 4 – 5 + 6 = 8
3 + 4 – 5 – 6 = -4
3 – 4 + 5 + 6 = 10
3 – 4 + 5 – 6 = -2
3 – 4 – 5 + 6 = 0
3 – 4 – 5 – 6 = -12
Observations:
All results are even numbers.
There are only eight possible combinations.
The results can be positive, negative, or zero.

Question 3.
Now, take four other consecutive numbers. Place the '+' and '-' signs as you have done before. Find out the results of each expression. What do you observe?
Solution:
Let us take numbers 5, 6, 7, 8:
Now,



5 + 6 + 7 + 8 = 26
5 + 6 + 7 – 8 = 10
5 + 6 – 7 + 8 = 12
5 + 6 – 7 – 8 = -4
5 – 6 + 7 + 8 = 14
5 – 6 + 7 – 8 = -2
5 – 6 – 7 + 8 = 0
5 – 6 – 7 – 8 = -16

Conclusion:

Regardless of the starting consecutive numbers, the same pattern of even results emerges when using all possible combinations of plus and minus signs. This is because the sum and difference of consecutive numbers will always result in an even number.

Question 4

Replace any negative sign in the expression $a + b - c - d$with a positive sign and find the difference between the two numbers.

Original expression:

$$a + b - c - d$$

Replace one negative sign by a positive sign.

Case 1

Replace $-c$by $+c$

New expression:

$$a + b + c - d$$

Difference:

$$(a + b + c - d) - (a + b - c - d)$$
$$= 2c$$

which is an even number.

Case 2

Replace $-d$by $+d$

New expression:

$$a + b - c + d$$

Difference:

$$(a + b - c + d) - (a + b - c - d)$$
$$= 2d$$

which is also even.

Observation

The difference is always twice a number.

Hence, the difference is always **even**.

Answer

$$\boxed{\text{The difference is always an even number.}}$$

BREAKING EVEN –

Question 1

We know how to identify even numbers. Without computing them, find which of the following arithmetic expressions are even.

(i) 43 + 37

43 is odd and 37 is odd.

Rule:

$$\text{Odd} + \text{Odd} = \text{Even}$$

Therefore,

$$43 + 37$$

is **Even**.

(ii) 672 − 348

672 is even and 348 is even.

Rule:

$$\text{Even} - \text{Even} = \text{Even}$$

Therefore,

$$672 - 348$$

is **Even**.

(iii) 4 × 347 × 3

4 is even.

Rule:

If one factor is even, the product is even.

Therefore,

$$4 \times 347 \times 3$$

is **Even**.

(iv) 708 − 477

708 is even and 477 is odd.

Rule:

$$\text{Even} - \text{Odd} = \text{Odd}$$

Therefore,

$$708 - 477$$

is **Odd**.

(v) 809 + 214

809 is odd and 214 is even.

Rule:

$$\text{Odd} + \text{Even} = \text{Odd}$$

Therefore,

$$809 + 214$$

is **Odd**.

(vi) 119 × 303

119 is odd and 303 is odd.

Rule:

$$\text{Odd} \times \text{Odd} = \text{Odd}$$

Therefore,

$$119 \times 303$$

is **Odd**.

(vii) 543 − 479

543 is odd and 479 is odd.

Rule:

$$\text{Odd} - \text{Odd} = \text{Even}$$

Therefore,

$$543 - 479$$

is **Even**.

(viii) 513^3

513 is odd.

Rule:

Odd × Odd × Odd = Odd

Therefore,

$$513^3$$

is **Odd**.

2. Using our understanding of how parity behaves under different operations, identify which of the following algebraic expressions give an even number for any integer values for the letter-numbers. (Page 115)

$2a + 2b$ | $3g + 5h$ | $4m + 2n$ | $2u - 4v$

$13k - 5k$ | $6m - 3n$ | $x^2 + 2$ | $b^2 + 1$ | $4k \times 3j$

1. 2a + 2b
2. 3g + 5h
3. 4m + 2n
4. 2u – 4v
5. 13k – 5k
6. 6m – 3n
7. $x^2 + 2$
8. $b^2 + 1$
9. 4k × 3j

Solution:

1. Analyse the expression 2a + 2b

2a is even because it's a multiple of 2.
2b is even because it's a multiple of 2.
The sum of two even numbers is even, so 2a + 2b is always even.

2. Analyse the expression 3g + 5h
If g is odd and h is odd, then 3g is odd and 5h is odd.
The sum of two odd numbers is even,
e.g., $3 \times 1 + 5 \times 1 = 8$.
If g is even and h is even, then 3g and 5h are even.
The sum of two even numbers is even,
e.g., $3 \times 2 + 5 \times 2 = 16$.
If g is odd and h is even, then 3g is odd and 5h is even.
The sum of an odd and an even number is odd,
e.g., $3 \times 1 + 5 \times 2 = 13$
Thus, 3g + 5h is not always even.

3. Analyse the expression 4m + 2n
4m is even because it's a multiple of 2.
2n is even because it's a multiple of 2.
The sum of two even numbers is even, so 4m + 2n is always even.

4. Analyse the expression 2u – 4v
2u is even because it's a multiple of 2.
4v is even because it's a multiple of 2.
The difference of two even numbers is even, so 2u – 4v is always even.

5. Analyse the expression 13k – 5k
This simplifies to 8k.
8k is even because it's a multiple of 2.
Thus, 13k – 5k is always even.

6. Analyse the expression 6m – 3n
6m is always even.
3n can be odd (if n is odd) or even (if n is even).
If n is odd, 3n is odd, and the difference of an even and an odd number is odd,
e.g. $6 \times 1 - 3 \times 1 = 3$.
Thus, 6m – 3n is not always even.

7. Analyse the expression $x^2 + 2$
x^2 is odd or even, both.
If x^2 is even, $x^2 + 2$ becomes even.
If x^2 is odd, $x^2 + 2$ becomes odd.
Thus, $x^2 + 2$ is not always even.

8. If b is even, b2 is even, and $b^2 + 1$ odd,
e.g., $2^2 + 1 = 5$.
If b is odd, b^2 is odd, and $b^2 + 1$ is even,
e.g, $3^2 + 1 = 10$.
Thus, $b^2 + 1$ is not always even.

9. Analyse the expression 4k × 3j
This simplifies to 12kj.
12kj is even because it's a multiple of 2.
Thus, 4k + 3j is always even.

3. Similarly, determine and explain which of the other expressions always give even numbers. Write a couple of examples and non-examples, as appropriate, for each expression.

Given expressions:

(v) $\mathbf{13k - 5k}$

$$13k - 5k = 8k$$

Since 8 is even, $8k$is always even.

Examples

- $k = 2 \Rightarrow 16$
- $k = 5 \Rightarrow 40$

Non-example: None (always even)

(vi) $\mathbf{6m - 3n}$

$$6m - 3n = 3(2m - n)$$

This is not always even.

Examples (Even)

- $m = 2, n = 2 \Rightarrow 12 - 6 = 6$

Non-examples (Odd)

- $m = 1, n = 1 \Rightarrow 6 - 3 = 3$

Hence **not always even**.

(vii) $x^2 + 2$

Not always even.

Examples (Even)

- $x = 2 \Rightarrow 4 + 2 = 6$

Non-examples (Odd)

- $x = 1 \Rightarrow 1 + 2 = 3$

Hence **not always even**.

(viii) $b^2 + 1$

Not always even.

Examples (Even)

- $b = 1 \Rightarrow 1 + 1 = 2$

Non-examples (Odd)

- $b = 2 \Rightarrow 4 + 1 = 5$

Hence **not always even**.

(ix) $4k \times 3j$

$$4k \times 3j = 12kj$$

Since 12 is even, the product is always even.

Examples

- $k = 1, j = 2 \Rightarrow 24$
- $k = 3, j = 1 \Rightarrow 36$

Non-example: None (always even)

---

4. Write a few algebraic expressions which always give an even number.

Examples:

$$2n$$
$$4m + 2$$
$$6x$$
$$8p - 4$$
$$10a + 6$$
$$2(b + c)$$
$$12k$$
$$14y + 8$$

Reason

Each expression contains a factor of 2 (or can be written as a multiple of 2), so it always produces an **even number**.

Always Even Expressions: $2n,\ 4m + 2,\ 6x,\ 8p - 4,\ 10a + 6,\ 2(b + c),\ 12k,\ 14y + 8$

In Text

Given:

$$4p \text{ and } (4q + 2)$$

Algebra

$$4p + (4q + 2)$$
$$= 4p + 4q + 2$$
$$= 4(p + q) + 2$$

Since $4(p + q)$is divisible by 4, the expression always leaves a remainder of **2** when divided by 4.

Examples

p	q	Expression
1	2	4(1)+(4×2+2)=14
2	3	8+14=22
4	5	16+22=38

All these numbers leave remainder **2** when divided by 4.

What Remains?

Question

Find a number that has a remainder of 3 when divided by 5. Write more such numbers.

Solution

A number leaves remainder 3 on division by 5 if it is of the form

$$5n + 3$$

where $n$is a whole number.

Examples

For $n = 0$

$$5(0) + 3 = 3$$

For $n = 1$

$$5(1) + 3 = 8$$

For $n = 2$

$$5(2) + 3 = 13$$

For $n = 3$

$$5(3) + 3 = 18$$

For $n = 4$

$$5(4) + 3 = 23$$

Answer

$$\boxed{3,\ 8,\ 13,\ 18,\ 23,\ 28,\ 33,\ 38, \ldots}$$

All these numbers leave remainder **3** when divided by **5**.

Verification

$$3 = 5 \times 0 + 3$$
$$8 = 5 \times 1 + 3$$
$$13 = 5 \times 2 + 3$$
$$18 = 5 \times 3 + 3$$

Hence each number leaves remainder 3.

Figure It Out

Question 1.
The sum of four consecutive numbers is 34. What are these numbers?
Solution:
Let x, x + 1, x + 2, and x + 3 be the four consecutive numbers, respectively.
$\therefore x + (x + 1) + (x + 2) + (x + 3) = 34$
$\Rightarrow 4x + 6 = 34$
$\Rightarrow 4x = 34 - 6$
$\Rightarrow 4x = 28$
$\Rightarrow x = 7$
$\therefore x + 1 = 7 + 1 = 8$
$x + 2 = 7 + 2 = 9$
and $x + 3 = 7 + 3 = 10$
Thus, the four consecutive numbers are 7, 8, 9, and 10.

Question 2.
Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p.
Solution:
Given p is the greatest of five consecutive numbers.
The other four numbers in terms of p are $(p - 1)$, $(p - 2)$, $(p - 3)$, and $(p - 4)$.
$p - 1$ is the second largest number
$p - 2$ is the third largest number
$p - 3$ is the second smallest number
$p - 4$ is the smallest number
$\therefore p > (p - 1) > (p - 2) > (p - 3) > (p - 4)$.

Question 3.
For each statement below, determine whether it is always true, sometimes true,

or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
(i) The sum of two even numbers is a multiple of 3.
(ii) If a number is not divisible by 18, then it is also not divisible by 9.
(iii) If two numbers are not divisible by 6, then their sum is not divisible by 6.
(iv) The sum of a multiple of 6 and a multiple of 9 is a multiple of 3.
(v) The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.
Solution:
(i) Sometimes true, the sum of two even numbers is a multiple of 3.
Examples:

- 2 + 4 = 6, 8 + 10 = 18, 14 + 16 = 30 are multiples of 3.
- 2 + 6 = 8, 4 + 10 = 14 are not multiples of 3.

(ii) Sometimes true, if a number is not divisible by 18, then it is also not divisible by 9.
Examples:

- 27 is not divisible by 18, but 27 is divisible by 9. True
- 40 is not divisible by 18, also it is not divisible by 9. False

(iii) Never true, if two numbers are not divisible by 6, then their sum is not divisible by 6.
Examples:

- 8 and 10 are not divisible by 6.
  The sum of two numbers = 8 + 10 = 18, is divisible by 6.
- 10 and 13 are not divisible by 6.
  The sum of 10 and 13 = 10 + 13 = 23, which is not divisible by 6.

(iv) Always true
Multiple of 6; 6m
Multiple of 9; 9n
6m + 9n = 3(2m + 3n)
Hence multiple of 3.

(v) Sometimes true, the sum of a multiple of 6 and a multiple of 3 is a multiple of 9.
Multiples of 6 are: 6, 12, 18, 24, 30,…
6 + 12 = 18 is a multiple of 9.
12 + 18 = 30 is not a multiple of 9.
18 + 24 = 42 is not a multiple of 9.

Sometimes true.
Multiples of 3 are: 3, 6, 9, 12, 15, 18,…
$3 + 6 = 9$ is a multiple of 9.
$6 + 9 = 15$ is not a multiple of 9.
Sometimes true.

Question 4.
Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.
Solution:
Here, Remainder = 2, Dividend = 3
∴ Number = (Quotient × Dividend) + Remainder = $(K \times 3) + 2$
where, K = 1, 2, 3,…..
Numbers = $1 \times 3 + 2 = 3 + 2 = 5$
Numbers = $2 \times 3 + 2 = 6 + 2 = 8$
Numbers = $3 \times 3 + 2 = 9 + 2 = 11$
Thus, 5, 8, and 11 are numbers that leave a remainder of 2 when divided by 3.
Algebraic expression = $3K + 2$
Here, Remainder = 2, dividend = 4
Number = $4K + 2$, where K = 1, 2, 3, 4, …
Numbers = $4 \times 1 + 2 = 4 + 2 = 6$
Numbers = $4 \times 2 + 2 = 8 + 2 = 10$
Numbers = $4 \times 3 + 2 = 12 + 2 = 14$
Algebraic expression = $4K + 2$
Thus, 6, 10, and 14 are numbers that leave a remainder of 2 when divided by 4.

Question 5.
“I hold some pebbles, not too many, when I group them in 3’s, one stays with me. Try pairing them up — it simply won’t do. A stubborn odd pebble remains in my view. Group them by 5, yet one’s still around, but grouping by seven, perfection is found. More than one hundred would be far too bold. Can you tell me the number of pebbles I hold?”

Solution:
The LCM of 3, 5, and 7 = 3 × 5 × 7 = 105 [∵ 3, 5, and 7 are prime numbers]
No. of pebbles = 105 + 1 = 106

Question 6.
Tathagat has written several numbers that leave a remainder of 2 when divided by 6. He claims, "If you add any three such numbers, the sum will always be a multiple of 6." Is Tathagat's claim true?
Solution:
The expression has been written by Tathagat = 6k + 2
where, k = 1, 2, 3, 4, 5, 6,…
6 × 1 + 2 = 8
6 × 2 + 2 = 14
6 × 3 + 2 = 20
6 × 4 + 2 = 26
The sum of three numbers
8 + 14 + 20 = 42, it is a multiple of 6.
14 + 20 + 26 = 60, it is a multiple of 6.
Yes, Tathagat's claim is true.

Question 7.
When divided by 7, the number 661 leaves a remainder of 3, and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 77? Show the solution both algebraically and visually.
(i) 4779 + 661
(ii) 4779 – 661

Solution:

Given, 661 = K × 7 + 3, where K = 1, 2, 3, 4, ...

and, 4779 = K × 7 + 5

Algebraic Method:

(i) 4779 + 661

4779 = (682 × 7) + 5

Remainder = 5

661 = (94 × 7) + 3

Remainder = 3

$\therefore \frac{4779+661}{7} = \frac{5+3}{7} = \frac{8}{7}$ = 1 = Remainder

(ii) 4779 − 661

$\therefore \frac{4779-661}{7} = \frac{5-1}{7} = \frac{8}{7}$ = 4 = Remainder

Visualization Method:

(i) 4779 + 661 = (682 × 7) + 5 + (94 × 7) + 3

= 7 × (682 + 94) + 5 + 3

= 7 × 776 + 8

= Divisible by 7 + $\frac{8}{7}$

= 1, Remainder

(ii) 4779 − 661 = (682 × 7) + 5 − (94 × 7) − 3

= 7 × (682 − 94) + 5 − 3

= 7 × 588 + 2

= Divisible by 7 + 2

= 2, Remainder

Question 8.

Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

Solution:

The expression of a number that leaves a remainder of 2 when divided by 3.

Number = 3K + 2

3 × 1 + 2 = 5

3 × 2 + 2 = 8

$3 \times 3 + 2 = 11$
$3 \times 4 + 2 = 14$
$3 \times 5 + 2 = 17$
$3 \times 6 + 2 = 20$
The expression of a number that leaves a remainder of 3 when divided by 4.
Number = 4K + 3
$4 \times 1 + 3 = 7$
$4 \times 2 + 3 = 11$
$4 \times 3 + 3 = 15$
$4 \times 4 + 3 = 19$
The expression of a number that leaves a remainder of 4 when divided by 5.
Number = 5K + 4
$5 \times 1 + 4 = 9$
$5 \times 2 + 4 = 14$
$5 \times 3 + 4 = 19$
$5 \times 4 + 4 = 24$
Smallest number = LCM of (3, 4, 5) – 1
= 60 – 1
= 59
59 is the smallest number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5.

IN TEXT

Question

Look at each of the following statements. Which are correct and why?

(i) If a number is divisible by 9, then the sum of its digits is divisible by 9.

Correct

Reason: A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

Example:

$$729$$

Sum of digits:

$$7 + 2 + 9 = 18$$

Since 18 is divisible by 9, 729 is divisible by 9.

(ii) If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Correct

This is the divisibility rule of 9.

Example:

$$459$$
$$4 + 5 + 9 = 18$$

18 is divisible by 9.

Therefore,

$$459 \div 9 = 51$$

So 459 is divisible by 9.

(iii) If a number is not divisible by 9, then the sum of its digits is not divisible by 9.

Correct

If the digit sum were divisible by 9, the number would also be divisible by 9.

Example:

$$125$$
$$1 + 2 + 5 = 8$$

8 is not divisible by 9.

Therefore, 125 is not divisible by 9.

(iv) If the sum of the digits of a number is not divisible by 9, then the number is not divisible by 9.

Correct

This follows directly from the divisibility rule of 9.

Example:

$$347$$
$$3 + 4 + 7 = 14$$

14 is not divisible by 9.

Therefore, 347 is not divisible by 9.

Conclusion

All four statements are **correct** because the divisibility rule for 9 states:

A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

Figure It Out

Question 1.
Find, without dividing, whether the following numbers are divisible by 9.
(i) 123
(ii) 405
(iii) 8888
(iv) 93547
(v) 358095
Solution:
If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.
(i) Sum of the digits = 1 + 2 + 3 = 6, is not divisible by 9.
Thus, 123 is not divisible by 9.

(ii) Sum of the digits = 4 + 0 + 5 = 9, is divisible by 9.
Thus, 405 is divisible by 9.

(iii) Sum of the digits = 8 + 8 + 8 + 8 = 32, is not divisible by 9.
Thus, 8888 is not divisible by 9.

(iv) Sum of the digits = 9 + 3 + 5 + 4 + 7 = 28, is not divisible by 9.
Thu,s 93547 is not divisible by 9.

(v) Sum of the digits = 3 + 5 + 8 + 0 + 9 + 5 = 30, is not divisible by 9.
Hence, 358095 is not divisible by 9.

Question 2.
Find the smallest multiple of 9 with no odd digits.
Solution:
Multiples of 9 = 9, 18, 27, 36,…, 288,………
The smallest multiple of 9 with an odd digit is 9.
The smallest multiple of 9 that can be formed by summing even digits is 18 (since 9 is odd).
Thus, the smallest multiple of 9 with no odd digits is 288.

Question 3.
Find the multiple of 9 that is closest to the number 6000.
Solution:
Given, 6000
Sum of the digits = 6 + 0 + 0 + 0 = 6
We know that, if the number is divisible by 9, then the sum of the digits is divisible by 9.
If we add 3 to the number 6000.
6000 + 3 = 6003, it is divisible by 3.
Thus, the multiple of 9 that is closest to the number is 6003.

Question 4.
How many multiples of 9 are there between the numbers 4300 and 4400?
Solution:
Solution:
The multiples of 9 are there between the numbers 4300 and 4400 are 4302, 4311, 4320,………, 4392
The number of multiples of 9 = $\frac{\text{Last term} - \text{First term}}{\text{Difference}} + 1$
$= \frac{4392-4302}{9} + 1$
$= \frac{90}{9} + 1$
= 10 + 1
= 11
Thus, the multiples of 9 are 11.

IN TEXT

Q1. Using these observations, can you tell whether 462 is divisible by 11?

For divisibility by 11:

Difference of the sums of alternate digits must be 0 or a multiple of 11.

For 462:

$$(4 + 2) - 6$$
$$6 - 6 = 0$$

Since 0 is a multiple of 11,

$$\boxed{462 \text{ is divisible by } 11}$$

Verification:

$$462 \div 11 = 42$$

Q2. What could be a general method or shortcut to check divisibility by 11?

Shortcut

1. Add the digits in alternate places.
2. Find the difference between the two sums.
3. If the difference is:
    - 0, or
    - 11, 22, 33, ... (a multiple of 11)

then the number is divisible by 11.

Example

For 847:

$$(8 + 7) - 4$$
$$15 - 4 = 11$$

Since 11 is a multiple of 11,

$$847 \text{ is divisible by } 11.$$

Q3. If this difference is 11 or a multiple of 11, what does that say about the remainder obtained when the number is divided by 11?

If the difference is 0 or a multiple of 11, then the number is exactly divisible by 11.

Hence,

$$\boxed{\text{Remainder } = 0}$$

Q4. Using this shortcut, find whether the following numbers are divisible by 11. If not, find the remainder.

(i) 158

$$(1+8)-5$$
$$9-5=4$$

4 is not a multiple of 11.

Therefore, 158 is **not divisible by 11**.

Remainder:

$$158 = 11 \times 14 + 4$$
$$\boxed{\text{Remainder } = 4}$$

(ii) 841

$$(8+1)-4$$
$$9-4=5$$

5 is not a multiple of 11.

Therefore, 841 is **not divisible by 11**.

$$841 = 11 \times 76 + 5$$
$$\boxed{\text{Remainder } = 5}$$

(iii) 481

$$(4 + 1) - 8$$
$$5 - 8 = -3$$

Difference = 3.

Not divisible by 11.

$$481 = 11 \times 43 + 8$$
$$\boxed{\text{Remainder } = 8}$$

(iv) 5529

$$(5 + 2) - (5 + 9)$$
$$7 - 14 = -7$$

Difference = 7.

Not divisible by 11.

$$5529 = 11 \times 502 + 7$$
$$\boxed{\text{Remainder } = 7}$$

(v) 90904

$$(9 + 9 + 4) - (0 + 0)$$
$$22 - 0 = 22$$

22 is a multiple of 11.

Therefore,

$$\boxed{90904 \text{ is divisible by } 11}$$

Remainder:

$$\boxed{0}$$

(vi) 857076

$$(8 + 7 + 7) - (5 + 0 + 6)$$
$$22 - 11 = 11$$

11 is a multiple of 11.

Therefore,

$$\boxed{857076 \text{ is divisible by } 11}$$

Remainder:

$$\boxed{0}$$

IN TEXT

Q1. Look at the following procedure.

For a number, place alternating $+$and $-$signs before the digits starting from the units digit and find the value.

A number is divisible by 11 if the result is:

- 0, or
- a multiple of 11.

Is this method similar to or different from the method for 9?

Answer:

It is different.

Divisibility by 9	Divisibility by 11
Add all digits.	Take alternating sum and difference of digits.
Example: $729 \rightarrow 7 + 2 + 9 = 18$	Example: $328105 \rightarrow -3 + 2 - 8 + 1 - 0 + 5 = -3$

Divisibility by 9	Divisibility by 11
If the sum is divisible by 9, the number is divisible by 9.	If the alternating sum is 0 or a multiple of 11, the number is divisible by 11.

Q2

Complete the table.

We use the divisibility rules.

Number	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
429714	Yes	Yes	No	No	Yes	No	No	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	Yes	No	No

Q.3 How can we find out if a number is divisible by 6?

A number is divisible by **6** if it is divisible by **both 2 and 3**.

Therefore, the divisibility rule for 6 is:

1. The number must be **even** (divisible by 2).
2. The sum of its digits must be divisible by **3**.

If both conditions are satisfied, the number is divisible by 6.

Examples

(i) 186

- Last digit = 6 (even) ✓
- Sum of digits = 1 + 8 + 6 = 15
- 15 is divisible by 3 ✓

Therefore,

$$\boxed{186 \text{ is divisible by } 6}$$

(ii) 225

- Last digit = 5 (not even) ✗

Therefore,

$$\boxed{225 \text{ is not divisible by } 6}$$

Q.4 Will checking divisibility by its factors 2 and 3 work? Verify −38, 225, 186, 64.

A number is divisible by 6 only if it is divisible by both 2 and 3.

(i) −38

- Even number ✓
- Sum of digits = 3 + 8 = 11
- 11 is not divisible by 3 ✗

$$\boxed{-38 \text{ is not divisible by } 6}$$

(ii) 225

- Last digit = 5, so not divisible by 2 ✗

- Sum of digits = 2 + 2 + 5 = 9 (divisible by 3) ✓

Since it is not divisible by 2,

$$\boxed{225 \text{ is not divisible by } 6}$$

(iii) 186

- Last digit = 6 (divisible by 2) ✓
- Sum of digits = 1 + 8 + 6 = 15
- 15 is divisible by 3 ✓

Therefore,

$$\boxed{186 \text{ is divisible by } 6}$$

Check:

$$186 \div 6 = 31$$

(iv) 64

- Last digit = 4 (divisible by 2) ✓
- Sum of digits = 6 + 4 = 10
- 10 is not divisible by 3 ✗

Therefore,

$$\boxed{64 \text{ is not divisible by } 6}$$

Conclusion

Yes, checking divisibility by **2 and 3** works because

$$6 = 2 \times 3$$

So a number is divisible by 6 **if and only if it is divisible by both 2 and 3**

DIGITAL ROOTS

1. What property do you think this digital root will have?

A number and its digital root leave the **same remainder when divided by 9**.

Therefore:

- If a number is divisible by 9, its digital root is **9**.
- If a number is not divisible by 9, its digital root gives the remainder when divided by 9.

---

2. Between 600 and 700, which numbers have the digital root

(i) 5

600 has digital root:

$$6 + 0 + 0 = 6$$

Digital roots increase cyclically:

6, 7, 8, 9, 1, 2, 3, 4, 5, ...

Hence numbers with digital root 5 are:

$$\boxed{608,\ 617,\ 626,\ 635,\ 644,\ 653,\ 662,\ 671,\ 680,\ 689,\ 698}$$

(ii) 7

$$\boxed{601,\ 610,\ 619,\ 628,\ 637,\ 646,\ 655,\ 664,\ 673,\ 682,\ 691,\ 700}$$

(iii) 3

$$\boxed{606,\ 615,\ 624,\ 633,\ 642,\ 651,\ 660,\ 669,\ 678,\ 687,\ 696}$$

---

3. Write the digital roots of any 12 consecutive numbers. What do you observe?

Take 25 to 36.

Number	Digital Root
25	7
26	8
27	9
28	1
29	2
30	3
31	4
32	5
33	6
34	7
35	8
36	9

Observation

Digital roots repeat in the cycle

$$\boxed{1, 2, 3, 4, 5, 6, 7, 8, 9}$$

again and again.

---

4. Find the digital roots of some consecutive multiples of

(i) 3

Take 3, 6, 9, 12, 15, 18, 21, 24

Digital roots:

$$3,\ 6,\ 9,\ 3,\ 6,\ 9,\ 3,\ 6$$

Observation

Digital roots of multiples of 3 are always

$$\boxed{3,\ 6, \text{or } 9}$$

(ii) 4

Take 4, 8, 12, 16, 20, 24, 28, 32

Digital roots:

$$4,\ 8,\ 3,\ 7,\ 2,\ 6,\ 1,\ 5$$

Observation

No fixed pattern of only one type of digital root.

(iii) 6

Take 6, 12, 18, 24, 30, 36, 42, 48

Digital roots:

$$6,\ 3,\ 9,\ 6,\ 3,\ 9,\ 6,\ 3$$

Observation

Digital roots repeat as

$$\boxed{6,\ 3,\ 9}$$

---

5. What are the digital roots of numbers that are 1 more than a multiple of 6? What do you notice?

Take:

$$7,\ 13,\ 19,\ 25,\ 31,\ 37,\ 43$$

Digital roots:

$$7, 4, 1, 7, 4, 1, 7$$

Observation

The digital roots repeat in the pattern

$$\boxed{1, 4, 7}$$

for numbers that are one more than a multiple of 6.

---

6. Puzzle

I'm made of digits, each tiniest and odd.
No shared ground with root #1.
My digits count, their sum, my root—
All point to one bold number's pursuit.
The largest odd single-digit I proudly claim.

Solution

The largest odd single-digit number is

$$\boxed{9}$$

So:

- Number of digits = 9
- Sum of digits = 9
- Digital root = 9
- Each digit is the smallest odd digit = 1

Hence the number is

$$\boxed{111111111}$$

Check:

- 9 digits ✓
- Sum = $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9$✓
- Digital root = 9 ✓

Answer

$$\boxed{111111111}$$

Name of the number: Ninety-nine million, nine hundred ninety-nine thousand, nine hundred ninety-nine? Wait, that's not correct.

$111111111$is read as:

$$\boxed{\text{One hundred eleven million, one hundred eleven thousand, one hundred eleven}}$$

Figure It Out

Q1

The digital root of an 8-digit number is 5. What will be the digital root of 10 more than that number?

Let the number be N.

Digital root of N= 5.

Adding 10 increases the digital root by 1 (since digital root of 10 is 1).

$$5 + 1 = 6$$

Answer

$$\boxed{6}$$

Figure It Out – Q2

Write any number. Generate a sequence by repeatedly adding 11. What will be the digital roots? Share your observations.

Take 25 as an example.

$$25,\ 36,\ 47,\ 58,\ 69,\ 80,\ 91, \ldots$$

Digital roots:

$$\begin{aligned} &2+5=7\\ &3+6=9\\ &4+7=11\to 2\\ &5+8=13\to 4\\ &6+9=15\to 6\\ &8+0=8\\ &9+1=10\to 1 \end{aligned}$$

Digital roots:

$$7,\ 9,\ 2,\ 4,\ 6,\ 8,\ 1,\ldots$$

Observation

Since

$$11\equiv 2(\text{mod} 9)$$

the digital root increases by 2 each time (cyclically).

Figure It Out – Q3

What will be the digital root of

$$9a+36b+13?$$

Since

$$9a$$

and

$$36b$$

are multiples of 9, their digital roots are 9 (or 0 modulo 9).

Thus only 13 affects the digital root.

$$1 + 3 = 4$$

Answer

$$\boxed{4}$$

Figure It Out – Q4

(i) Relation between parity and digital root

Parity means evenness or oddness.

Examples:

Number	Digital Root
24	6
35	8
18	9
21	3

Observation:

- Even numbers may have odd or even digital roots.
- Odd numbers may have odd or even digital roots.

Conclusion

$$\boxed{\text{There is no fixed relation between parity and digital root.}}$$

(ii) Relation between digital root and remainder when divided by 3 or 9

Examples:

Number	Digital Root	Remainder on division by 9
25	7	7

Number	Digital Root	Remainder on division by 9
38	2	2
45	9	0

Observation

The digital root gives the same remainder as the number when divided by 9 (except digital root 9 corresponds to remainder 0).

Conclusion

$$\boxed{\text{Digital root helps determine divisibility by 9 and 3.}}$$

DIGITS IN DISGUISE

Q1 (i)

$$\begin{array}{l} A1 \\ +1B \\ = BO \end{array}$$

Let the units digit be O.

Units column:

$$1 + B = O$$

Tens column:

$$A + 1 = B$$

Try $A = 2$

$$B = 3$$

Then

$$21 + 13 = 34$$

Thus

$$B = 3,\ O = 4$$

Answer

$$\boxed{21 + 13 = 34}$$

Q1 (ii)

$$AB + 37 = 6A$$

Let

$$AB = 10A + B$$

Then

$$10A + B + 37 = 60 + A$$
$$9A + B = 23$$

Try digits:

$$A = 2$$
$$B = 5$$

Check:

$$25 + 37 = 62$$

Answer

$$\boxed{25 + 37 = 62}$$

Q1 (iii)

$$ON + ON + ON = PO$$
$$3(10O + N) = 10P + O$$

Try $O = 1$

$$3(10 + N) = 10P + 1$$

For $N = 7$,

$$17 \times 3 = 51$$

Answer

$$\boxed{17 + 17 + 17 = 51}$$

$(O = 1,\ N = 7,\ P = 5)$

Q1 (iv)

$$\begin{aligned}&QR + QR + QR = PRR\\&3(10Q + R) = 100P + 11R\\&30Q - 8R = 100P\end{aligned}$$

Try $P = 1$

$$15Q - 4R = 50$$

$Q = 6,\ R = 10$not possible.

$$Q = 4,\ R = 5$$

Check:

$$45 + 45 + 45 = 135$$

Answer

$$\boxed{45 + 45 + 45 = 135}$$

$(Q = 4,\ R = 5,\ P = 1)$

2. Solve the following

(i) $UT \times 3 = PUT$

Let $UT = 10U + T$

$$3(10U + T) = 100P + 10U + T$$

Since the product is a 3-digit number beginning with P, try $UT = 50$:

$$50 \times 3 = 150$$

This matches the form PUT.

Therefore,

$$U = 5, T = 0, P = 1$$

Answer

$$\boxed{50 \times 3 = 150}$$

(ii) $AB \times 5 = BC$

Let $AB = 10A + B$.

Since multiplying by 5 gives a 2-digit number BC,

try 13:

$$13 \times 5 = 65$$

This fits BC.

Thus,

$$A = 1, B = 3, C = 5$$

Answer

$$\boxed{13 \times 5 = 65}$$

(iii) $L2N \times 2 = 2NP$

Units digit:

$$2 \times N = P$$

Try $N = 4$,

$$224 \times 2 = 448$$

This fits the pattern $2NP$.

Hence,

$$L = 2, N = 4, P = 8$$

Answer

$$\boxed{224 \times 2 = 448}$$

(iv) $XY \times 4 = ZX$

Since the product is a 2-digit number ending in X,

try 13:

$$13 \times 4 = 52$$

Here

$$X = 1, Y = 3, Z = 5$$

Answer

$$\boxed{13 \times 4 = 52}$$

(v) $PP \times QQ = PRP$

$$PP = 11P, QQ = 11Q$$

So,

$$PP \times QQ = 121PQ$$

Try $P = 1$, $Q = 2$:

$$11 \times 22 = 242$$

which is of the form PRP.

Thus,

$$P = 2, R = 4, Q = 1$$

Check:

$$22 \times 11 = 242$$

Answer

$$\boxed{22 \times 11 = 242}$$

(vi) $JK \times 6 = KKK$

$$\begin{aligned}
&(10J + K) \times 6 = 111K \\
&60J + 6K = 111K \\
&60J = 105K \\
&4J = 7K
\end{aligned}$$

Smallest digit solution:

$$J = 7, K = 4$$

Check:

$$74 \times 6 = 444$$

Answer

$$\boxed{74 \times 6 = 444}$$

Figure It Out

Question 1.
If 31z5 is a multiple of 9, where z is a digit, what is the value of z? Explain why there are two answers to this problem.
Solution:
Here, 31z5
Sum of the digits = 3 + 1 + z + 5 = 9 + z
(9 + z) should be divisible by 9.
z = 0, 3105 is divisible by 9.
z = 9, 3195 is also divisible by 9.
∴ z = 0 or 9
There are two answers to this problem because, excluding z, the sum of the digits is divisible by 9.

Question 2.
"I take a number that leaves a remainder of 8 when divided by 12.1 take another number, which is 4 short of a multiple of 12. Their sum will always be a multiple of 8", claims Snehal. Examine his claim and justify your conclusion.
Solution:
A number that leaves a remainder of 8.
when divided by 12: 12k + 8, where k ≥ 1.
Also, another number 4 short of a multiple of 12: 12k – 4

Question 3.
When is the sum of two multiples of 3, a multiple of 6, and when is it not? Explain the different possible cases, and generalise the pattern.
Solution:
Multiples of 3 are: 3, 6, 9, 12, 15, 18,……….
3 + 6 = 9, not a multiple of 6.
6 + 9 = 15, not a multiple of 6.
3 + 9 = 12, multiple of 6.
6 + 12 = 18, multiple of 6.
There are two possible cases.

- If both numbers are odd, then the sum is a multiple of 6.
- If both numbers are even, then the sum is a multiple of 6.

Question 4.
Sreelatha says, “I have a number that is divisible by 9. If I reverse its digits, it will still be divisible by 9”.
(i) Examine if her conjecture is true for any multiple of 9.
(ii) Are any other digit shuffles possible such that the number formed is still a multiple of 9?
Solution:
Consider a number that is divisible by 9 = 72
We know that,
If the sum of the digits is divisible by 9, then the number is divisible by 9.
If its digits are reversed
27 = 2 + 7 = 9, it is also divisible by 9.
(i) True
(ii) Yes, any other digit shuffle is possible that the number is still a multiple of 9.

Question 5.
If 48a23b is a multiple of 18, list all possible pairs of values for a and b.
Solution:
Given by question,
48a23b is a multiple of 18.
As we know that,
If the number is a multiple of 18, then it is also a multiple of 2 and 9.
∴ 48a23b
Sum of the digits = 4 + 8 + a + 2 + 3 + b = 17 + a + b

Case 1: Put a = 1 and b = 0
481230, it is possible values of a and b.
Sum = 18, it is divisible by 9.

Case 2: Put a = 4 and b = 6
484236
Sum = 17 + 10 = 27, it is divisible by 9.
Thus, the possible values of a and 6 are a = 1 and b = 0, a = 4 and b = 6; there are two possible cases.

Question 6.
If 3p7q8 is divisible by 44, list all possible pairs of values for p and q.
Solution:
Given by question, 3p7q8 is divisible by 44.
As we know, if a number is divisible by 44, then it is also divisible by 4 and 11.

∴ 3p7q8
Case 1: Put p = 1 and q = 0
37708 is divisible by 4 and 11, then it is also divisible by 44.

Case 2: Put p = 5 and q = 2
35728 is divisible by 4 and 11, then it is also divisible by 44.

Case 3: Put p = 3 and q = 4
33748 is divisible by 4 and 11, then it is also divisible by 44.

Case 4: Put p = 1 and q = 6
31768 is divisible by 4 and 11, then it is also divisible by 11.
Thus, (p = 7, q = 0), (p = 5, q = 2), (p = 3, q = 4), and (p = 1 and q = 6) are the possible pairs of values for p and q.

Question 7.
Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4. Are there more such numbers? How often do they occur?
Solution:
Let x, x + 1 and (x + 2) be the three numbers
Put x = 2, ⇒ 2, 3, 4
Put x = 14, ⇒ 14, 15, 6
Put x = 26, ⇒ 26, 27, 28
Put x = 38, ⇒ 38, 39, 40
Thus, the three consecutive numbers are (14, 15, 16),
Put x = 26, ⇒ 26, 27, 28
(26, 27, 28) and (38, 39, 40)
There are infinite numbers, spaced apart by 12.

Question 8.
Write five multiples of 36 between 45,000 and 47,000. Share your approach with the class.
Solution:
We know that if a number is a multiple of 36, then it is also a multiple of 4 and 9.
45000
Last two digits = 00, it is divisible by 4.
Sum of the digits = 4 + 5 + 0 + 0 + 0 = 9, it is also divisible by 9.
Thus, 45000 is completely divisible by 36.
The five multiples of 36 between 45,000 and 47,000.
(45,000 + 36), (45,000 + 2 × 36), (45,000 + 3 × 36), (45,000 + 4 × 36) and

(45,000 + 5 × 36)
i.e., 45,036, 45,072, 45,108, 45,144, and 45,180.

Question 9.
The middle number in the sequence of 5 consecutive even numbers is 5p. Express the other four numbers in sequence in terms of p.
Solution:
Given the middle number in the sequence of 5 consecutive even numbers 5p.
The other four numbers in the sequence in terms of p are 5p – 4, 5p – 2, 5p + 2, 5p + 4
Hence, the other four numbers in sequence are p, 3p, 7p, and 9p.

Question 10.
Write a 6-digit number that is divisible by 15, such that when the digits are reversed, it is divisible by 6.
Solution:
We know that if the number is divisible by 3 and 5, then it is also divisible by 15.
Consider the number 643215.
Sum of the digits = 6 + 4 + 3 + 2 + 1 + 5 = 21, which is divisible by 3.
Thus, 643215 is divisible by 3.
One’s place = 5, it is also divisible by 5.
Hence, 643215 is divisible by 15.
One’s place is not 0, because the digits are reversed, it becomes a 5-digit number.
Lakhs place is always taken as an even number.
Reversed the digits:
512346
One’s place = 6, 512346 is divisible by 2.
Sum of the digits = 5 + 1 + 2 + 3 + 4 + 6 = 21.
It is also divisible by 3.
Hence, 512346 is divisible by 6.

Question 11.
Deepak claims, “There are some multiples of 11 which, when doubled, are still multiples of 11. But other multiples of 11 don’t remain multiples of 11 when doubled”. Examine if his conjecture is true; explain your conclusion.
Solution:
The multiples of 11 are: 11, 22, 33, 44, 55,…
When doubled, 22, 44, 66, 88, 110,……

i.e. (11) × 2, 11 × 4, 11 × 6, 11 × 8, 11 × 10,…. are also multiples of 11.
False, if multiples of 11 are doubled, then the multiples of 11 are these numbers.

Question 12.
Determine whether the statements below are ‘Always True’, ‘Sometimes True’, or ‘Never True’. Explain your reasoning.
(i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9.
(ii) The sum of three consecutive even numbers will be divisible by 6.
(iii) If abcdef is a multiple of 6, then badcef will be a multiple of 6.
(iv) 8(7b – 3) – 4(11b + 1) is a multiple of 12.
Solution:
(i) Always True,
The multiple of 6 can be written as 6a, where a is an integer.
The multiple of 3 can be written as 3b, where b is an integer.
∴ Product = (6a) × (3b) = 18(ab) is a multiple of 9.

(ii) Always True,
The sum of three consecutive even numbers will be divisible by 6.
For example 2 + 4 + 6 = 12, 4 + 6 + 8 = 18, 6 + 8 + 10 = 24, 8 + 10 + 12 = 30,…
These numbers are divisible by 6.

(iii) Always True, because one’s place does not change.

(iv) Sometimes true,
Conclusion:
8(7 × 1 – 3) – 4(11 × 1 + 1) = -16, not divisible by 12.
8(7 × 10 – 3) – 4(4 × 10 + 1) = 536 – 164 = 372, divisible by 12.

Question 13.
Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.
Solution:
Let the three numbers be n_1, n_2, and n_3.
Let their remainders when divided by 3 be r_1, r_2, and r_3.
The sum $n_1 + n_2 + n_3$ is divisible by 3 if and only if $r_1 + r_2 + r_3$ is divisible by 3.
Case 1: All remainders are 0.
$r_1 = 0$, $r_2 = 0$, $r_3 = 0$
Sum of remainders = 0 + 0 + 0 = 0, which is divisible by 3.

Case 2: All remainders are 1.
$r_1 = 1, r_2 = 1, r_3 = 1$
Sum of remainders = 1 + 1 + 1 = 3, which is divisible by 3.

Case 3: All remainders are 2.
$r_1 = 2, r_2 = 2, r_3 = 2$
Sum of remainders = 2 + 2 + 2 = 6, which is divisible by 3.

Case 4: One remainder is 0, one is 1, and one is 2.
$r_1 = 0, r_2 = 1, r_3 = 2$ (in any order).
Sum of remainders = 0 + 1 + 2 = 3, which is divisible by 3.
The sum of three numbers is divisible by 3 if and only if all three numbers have the same remainder when divided by 3, or if they all have different remainders when divided by 3.

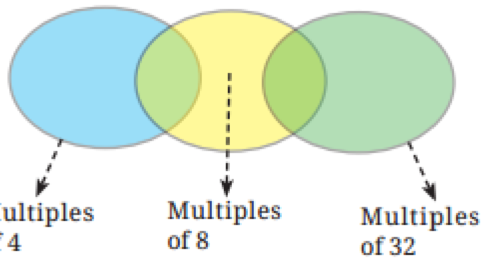
Question 14.
Is the product of two consecutive integers always a multiple of 2? Why? What about the product of these consecutive integers? Is it always a multiple of 6? Why or why not? What can you say about the product of 4 consecutive integers? What about the product of five consecutive integers?
Solution:
Yes, the product of two consecutive integers is always a multiple of 2.
1 × 2 = 2, 2 × 3 = 6, 5 × 6 = 30, 10 × 11 = 110, and so on.
Since we know that multiplying by an odd number and an even number is always an even number.
No, it is not always a multiple of 6.
1 × 2 = 2, 4 × 5 = 20, 7 × 8 = 56
Since it is not divisible by 6.
The product of 4 consecutive integers
2 × 3 × 4 × 5 = 120,
4 × 5 × 6 × 7 = 840,
5 × 6 × 7 × 8 = 1680
We can say that the product of 4 consecutive integers, divisible by 12.
The product of five consecutive integers is:
1 × 2 × 3 × 4 × 5 = 120,
2 × 3 × 4 × 5 × 6 = 720,
3 × 4 × 5 × 6 × 7 = 2520
Hence, we can say that the product of five consecutive integers is always divisible by 24.

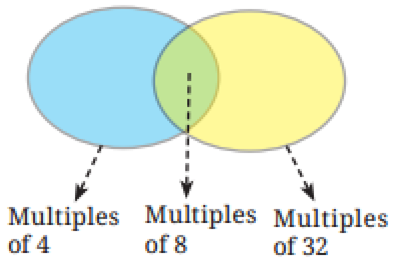
Question 15.
Solve the cryptarithms

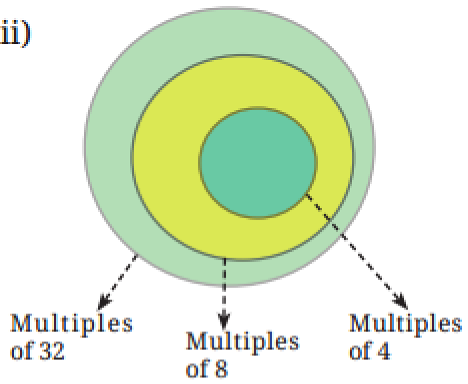
(i) EF × E = GGG
(ii) WOW × 5 = MEOW
Solution:
(i) This means a 2-digit number multiplied by 5 gives a 3-digit number.
2-digit number = 20, 21,…., 99
37 × 3 = 111, all conditions are satisfied.

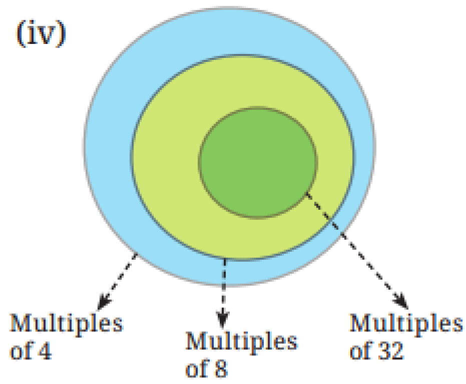
(ii) This means a 3-digit number multiplied by 5 gives 4-digit numbers.
Pick 3-digit number = 200, 201,…., 999
525 × 5 = 2625

Question 16.
Which of the following Venn diagrams captures the relationship between the multiples of 4, 8, and 32?

(i)


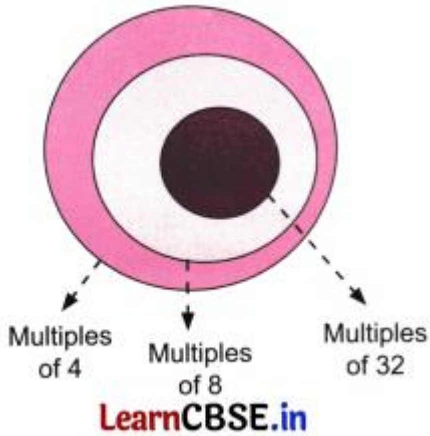
(ii)


(iii)


(iv)


Solution:
(iv) Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64,…
Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64,….
Multiples of 32 are: 32, 64, 96, 128,…
The Venn diagram captures the relationship between the multiples of 4, 8, and

32:



EXAM TIME

A. Multiple Choice Questions

1. Which of the following is not a sum of consecutive numbers?

- 12 = 3 + 4 + 5 ✓
- 9 = 4 + 5 ✓
- 11 = 5 + 6 ✓
- 16 cannot be expressed as a sum of two or more consecutive natural numbers.

Answer: (b) 16

---

2. If x, y, z and w are four consecutive numbers and '+' or '−' signs are placed between them, then the number of possible expressions is

There are 3 gaps between 4 numbers.

Each gap can have either + or −.

$$2^3 = 8$$

Answer: (d) 8

---

3. The expression $119 \times 332 \times 6$is a/an

$$119 = 7 \times 17$$
$$332 = 2 \times 2 \times 83$$
$$6 = 2 \times 3$$

Product has many factors.

Hence it is neither prime nor odd.

Also contains factor 2, so it is even.

Answer: (a) Even number

---

4. Which expression always gives an even number?

(a) $4a + 3b$→ not always even

(b) $a^2 + 1$→ not always even

(c) $3x + 5y$→ not always even

(d) $2x + 2y = 2(x + y)$→ always divisible by 2

Answer: (d) $2x + 2y$

---

5. The sum of two multiples of 4 is always a multiple of

Let numbers be $4a$and $4b$.

$$4a + 4b = 4(a + b)$$

Hence always a multiple of 4.

Answer: (d) 4

---

6. Which number is divisible by 8?

Check last 3 digits.

- 293 ✗
- 1205 → 205 not divisible by 8 ✗
- 1648 → 648 ÷ 8 = 81 ✓

- 2003 → 003 = 3 ✗

Answer: (c) 1648

---

7. If 7254*98 is divisible by 22, then * = ?

Divisible by 22 ⇒ divisible by 2 and 11.

For divisibility by 11:

$$(8 + * + 5 + 7) - (9 + 4 + 2)$$
$$(20 + *) - 15$$
$$* = ?$$

Difference must be multiple of 11.

Only * = 6 gives:

$$26 - 15 = 11$$

Answer: (c) 6

---

8. Smallest value of *() so that 653()*47 is divisible by 11

$$(7 + * + 5) - (4 + 3 + 6)$$
$$(12 + *) - 13$$
$$* = 1$$

Difference = 0.

Answer: (d) 1

---

9. 116 and 365 are divisible by

116:

$$116 \div 29 = 4$$

365:

$$365 \div 73 = 5$$

Neither 5 nor 7 divides both.

Answer: (d) None of these

---

10. 1265 is divisible by

Ends in 5.

Divisible by 5 only.

Not by 2, 10 or 11.

Answer: None of the given options (printing error in book)

---

11. Which numbers are divisible by 4?

- 4096 ✓
- 21084 ✓
- 31795012 ✓

All satisfy divisibility rule of 4.

Answer: (d) All of these

---

12. Which number is divisible by 9?

- 672 → sum = 15 ✗
- 5652 → sum = 18 ✓
- 4847 → sum = 23 ✗

Answer: (b) 5652

---

13. Which number is divisible by 6?

Need divisibility by 2 and 3.

- 297143 ✗ odd
- 1790184 → even, digit sum = 30 ✓

- 291245 ✗ odd
- 1840791 ✗ odd

Answer: (b) 1790184

---

14. A number is divisible by 90 if

$$90 = 9 \times 10$$

Must be divisible by 9 and 10.

Also implies divisibility by 2,5,18,45.

Hence all conditions are correct.

Answer: (d) All of these

---

15. If a number is divisible by 5 and 6 then it may be divisible by

LCM(5,6)=30

Answer: (a) 30

---

16. If two numbers are divisible by 9, their sum is always divisible by

$$9a + 9b = 9(a + b)$$

Answer: (d) 9

---

17. Remainder when 579 is divided by 8

$$8 \times 72 = 576$$
$$579 - 576 = 3$$

Answer: (b) 3

---

18. Digital root of 9876

$$9 + 8 + 7 + 6 = 30$$
$$3 + 0 = 3$$

Answer: (c) 3

---

19. If $5A \times A = 399$

$$57 \times 7 = 399$$

Thus

$$A = 7$$

Answer: (c) 7

---

20. If $A \times 3 = 1A$

$$A \times 3 = 10 + A$$
$$2A = 10$$
$$A = 5$$

Check:

$$5 \times 3 = 15$$

Answer: (b) 5

Fill in the Blanks

1. The expression $115 \times 707$is an ______ number.

707 is odd and 115 is odd.

Odd × Odd = Odd

Answer: odd

---

2. The sum of two even numbers that are not multiples of 4 is a multiple of ______.

Examples:

$$2 + 6 = 8$$
$$10 + 14 = 24$$
$$18 + 22 = 40$$

Each answer is divisible by 4.

Answer: 4

---

3. 3134673 is divisible by 3 and ______.

Digit sum:

$$3 + 1 + 3 + 4 + 6 + 7 + 3 = 27$$

27 is divisible by 9.

Hence the number is divisible by both 3 and 9.

Answer: 9

4. If 1x34 is divisible by 9, then x is equal to ______.

For divisibility by 9, the sum of digits must be a multiple of 9.

$$1 + x + 3 + 4 = x + 8$$

Possible multiples of 9:

$$x + 8 = 9 \Rightarrow x = 1$$

Therefore,

x = 1

---

5. If 435 is divisible by 15, then it is divisible by 3 and ______.

A number divisible by **15** must be divisible by both **3 and 5**.

Therefore,

Answer: 5

---

6. The digital root of 98764 is ______.

Sum of digits:

$$9 + 8 + 7 + 6 + 4 = 34$$

Again,

$$3 + 4 = 7$$

Therefore,

Digital Root = 7

---

7. If $A3 \times 3 = BA9$, then A = ______ and B = ______.

Let the two-digit number $A3 = 10A + 3$.

Given:

$$(10A + 3) \times 3 = BA9$$

Check options:

If $A = 3$,

$$33 \times 3 = 99$$

Not of form BA9.

If $A = 4$,

$$43 \times 3 = 129$$

This is of the form BA9.

So,

$$BA9 = 129$$

Hence,

$$B = 1, A = 4$$

Answer: A = 4, B = 1

C. True / False

1. The sum of four consecutive numbers is always even.

Let numbers be:

$$n,\ n + 1,\ n + 2,\ n + 3$$

Sum:

$$4n + 6 = 2(2n + 3)$$

Always even.

True

---

2. The expression 319^3 is an even number.

319 is odd.

Odd × Odd × Odd = Odd

False

---

3. If 231x27 is divisible by 9, then the value of x is 0.

Digit sum:

$$2 + 3 + 1 + x + 2 + 7 = 15 + x$$

For divisibility by 9:

$$15 + x = 18$$
$$x = 3$$

Not 0.

False

---

4. If 57x is divisible by 4, then the possible value of x can be 2 or 6.

Last two digits are 7x.

72 and 76 are divisible by 4.

Hence x can be 2 or 6.

True

---

5. If a number $\boldsymbol{a}$is divisible by $\boldsymbol{b}$, then it must be divisible by each factor of $\boldsymbol{b}$.

Example:

24 divisible by 12.

Factors of 12 are 2, 3, 4, 6.

24 is divisible by all of them.

True

---

6. All numbers divisible by 4 may not be divisible by 8.

Example:

12 is divisible by 4 but not by 8.

True

---

7. Number of the form $\boldsymbol{3N+2}$will leave remainder 2 when divided by 3.

$$3N + 2 = 3(N) + 2$$

Clearly remainder = 2.

True

---

8. The digital root of 68321 is 3.

$$6 + 8 + 3 + 2 + 1 = 20$$
$$2 + 0 = 2$$

Digital root = 2

False

D. Match the Columns

Column I

(a) If $N \div 2$leaves a remainder 1, then one's digit of N is ______

A number leaving remainder 1 on division by 2 is odd.

Possible one's digits:

1, 3, 5, 7, 9

➞ Matches **(ii)**

(b) If 36D is a multiple of 3 and D is a digit, then the value of D is ______

For divisibility by 3:

$$3 + 6 + D = 9 + D$$

Since 9 is already divisible by 3, D can be:

0, 3, 6, 9

➞ Matches **(iv)**

(c) If $N \div 5$leaves a remainder 1, then one's digit of N is ______

Numbers leaving remainder 1 when divided by 5 end in:

1 or 6

➞ Matches **(i)**

(d) If D is a digit and 21D5 is divisible by 9, then D is ______

Digit sum:

$$2 + 1 + D + 5 = 8 + D$$

For divisibility by 9:

$$8 + D = 9 \text{ or } 18$$
$$D = 1 \text{ or } 10$$

10 is not a digit.

$$D = 1$$

➞ Matches **(iii)**

Answer

Column I	Column II
(a)	(ii)
(b)	(iv)
(c)	(i)
(d)	(iii)

E. Very Short Answer Type Questions

1. Write three consecutive numbers whose sum is 12.

Let the numbers be:

$$n, n + 1, n + 2$$
$$n + (n + 1) + (n + 2) = 12$$
$$3n + 3 = 12$$
$$3n = 9$$
$$n = 3$$

Numbers are:

$$\boxed{3,\ 4,\ 5}$$

---

2. If 325a is divisible by 3, find the minimum value of a.

Digit sum:

$$3 + 2 + 5 + a = 10 + a$$

For divisibility by 3:

$$10 + a$$

must be divisible by 3.

Smallest digit satisfying this:

$$a = 2$$

because

$$10 + 2 = 12$$

and 12 is divisible by 3.

$$\boxed{a = 2}$$

---

3. A 3-digit number 42x is divisible by 9. Find x.

Digit sum:

$$4 + 2 + x = 6 + x$$

For divisibility by 9:

$$6 + x = 9$$
$$x = 3$$
$$\boxed{x = 3}$$

---

4. Check the divisibility of 1052896 by 9.

Digit sum:

$$1 + 0 + 5 + 2 + 8 + 9 + 6 = 31$$

31 is not divisible by 9.

Hence

$$\boxed{\text{1052896 is not divisible by 9}}$$

---

5. A number is divisible by 20. By what other number will that number be divisible?

Since

$$20 = 2 \times 10 = 4 \times 5$$

Any multiple of 20 is divisible by:

$$\boxed{2, 4, 5, 10}$$

---

6. When $N \div 2$leaves remainder 1, what will be the one's digit of N?

Such numbers are odd.

Possible one's digits:

$$\boxed{1, 3, 5, 7, 9}$$

---

7. Find the digital root of 7694.

$$7 + 6 + 9 + 4 = 26$$
$$2 + 6 = 8$$

Digital root:

$$\boxed{8}$$

---

8. Find the values of A and B.

Given:

$$\begin{array}{c} A \\ A \\ +A \\ +A \\ BA \end{array}$$

So,

$$A + A + A + A = BA$$
$$4A = BA$$

A single digit satisfying this:

$$A = 5$$
$$4 \times 5 = 20$$

Hence:

$$\boxed{A = 5,\ B = 2}$$

F. Short Answer Type Questions

1. Which of the following expressions is/are even?

(i) 497 + 364

497 is odd and 364 is even.

Odd + Even = Odd

$$497 + 364 = 861$$

❌ Not even

(ii) 1229 − 871

Both numbers are odd.

Odd − Odd = Even

$$1229 - 871 = 358$$

✅ Even

Answer

$$\boxed{\text{Only (ii) is even}}$$

---

2. Check the divisibility of the following numbers by 10.

(i) 990

A number is divisible by 10 if its one's digit is 0.

One's digit = 0

✅ 990 is divisible by 10.

$$990 \div 10 = 99$$

(ii) 1043

One's digit = 3

❌ Not divisible by 10.

Answer

Number	Divisible by 10?
990	Yes
1043	No

---

3. If 21y5 is a multiple of 9, where y is a digit, what is the value of y?

For divisibility by 9:

$$2 + 1 + y + 5$$
$$= 8 + y$$

must be divisible by 9.

$$8 + y = 9$$
$$y = 1$$

Answer

$$\boxed{y = 1}$$

---

4. Suppose the division $N \div 5$leaves a remainder of 4 and the division $N \div 2$leaves a remainder of 1. What must be the one's digit of N?

From $N \div 5$remainder 4:

Possible one's digits:

$$4,\ 9$$

From $N \div 2$remainder 1:

Number must be odd.

Possible one's digits:

$$1,\ 3,\ 5,\ 7,\ 9$$

Common digit:

$$\boxed{9}$$

Answer

$$\boxed{9}$$

---

5. Find the values of A, B and C.

Given:

$$\begin{array}{r} 4A \\ +98 \\ CB3 \end{array}$$

Ones column

$$A + 8 = 3$$

A carry of 1 must be generated.

$$\begin{aligned} &A + 8 = 13 \\ &A = 5 \end{aligned}$$

Carry = 1

Tens column

$$4 + 9 + 1 = 14$$

So

$$B = 4$$

Carry = 1

Hundreds column

$$C = 1$$

Verification

$$45 + 98 = 143$$

Hence

$$C = 1,\ B = 4,\ A = 5$$

Answer

$$\boxed{A = 5,\ B = 4,\ C = 1}$$

---

6. Find the values of A and B.

Given:

$$\begin{array}{c} A1 \\ +1B \\ B0 \end{array}$$

Ones column

$$\begin{aligned} &1 + B = 10 \\ &B = 9 \end{aligned}$$

Carry = 1

Tens column

$$\begin{aligned} &A + 1 + 1 = B \\ &A + 2 = 9 \\ &A = 7 \end{aligned}$$

Verification

$$71 + 19 = 90$$

Correct.

Answer

$$\boxed{A = 7,\ B = 9}$$

G. Long Answer Type Questions

1. Using divisibility tests, determine which of the following numbers are divisible by 6.

A number is divisible by 6 if it is divisible by both 2 and 3.

(i) 297144

Divisible by 2?

Last digit = 4 (even)

✅ Yes

Divisible by 3?

$$2 + 9 + 7 + 1 + 4 + 4 = 27$$

27 is divisible by 3.

✅ Yes

Therefore,

$$\boxed{297144 \text{ is divisible by } 6}$$

(ii) 1258

Divisible by 2?

Last digit = 8

✅ Yes

Divisible by 3?

$$1 + 2 + 5 + 8 = 16$$

16 is not divisible by 3.

❌ No

Therefore,

$$\boxed{1258 \text{ is not divisible by } 6}$$

(iii) 4335

Divisible by 2?

Last digit = 5

❌ No

Therefore,

$$\boxed{4335 \text{ is not divisible by } 6}$$

(iv) 61233

Divisible by 2?

Last digit = 3

❌ No

Therefore,

$$\boxed{61233 \text{ is not divisible by } 6}$$

Answer

Only

$$\boxed{297144}$$

is divisible by 6.

---

2. Write a digit in the blank so that the number formed is divisible by 11.

(i) 92□389

Divisibility rule of 11:

Difference between the sum of alternate digits must be 0 or a multiple of 11.

Let the blank be x.

Digits:

9, 2, x, 3, 8, 9

Odd places:

$$9 + x + 8$$
$$= 17 + x$$

Even places:

$$2 + 3 + 9 = 14$$

Difference:

$$(17 + x) - 14 = x + 3$$

Must be 11.

$$x + 3 = 11$$
$$x = 8$$

Answer

$$\boxed{8}$$

Verification:

928389

$$(9 + 8 + 8) - (2 + 3 + 9) = 25 - 14 = 11$$

✓

(ii) 8□9484

Let blank = x

Digits:

8, x, 9, 4, 8, 4

Odd places:

$$8 + 9 + 8 = 25$$

Even places:

$$x + 4 + 4 = x + 8$$

Difference:

$$25 - (x + 8) = 17 - x$$

Must be 11.

$$17 - x = 11$$
$$x = 6$$

Answer

$$\boxed{6}$$

Verification:

869484

$$(8 + 9 + 8) - (6 + 4 + 4) = 25 - 14 = 11$$

✓

---

3. If 31z5 is a multiple of 3, where z is a digit, what might be the values of z?

For divisibility by 3:

$$3 + 1 + z + 5$$

$$= 9 + z$$

must be divisible by 3.

Since 9 is already divisible by 3,

$z$must be a multiple of 3.

Possible values:

$$\boxed{0,\ 3,\ 6,\ 9}$$

---

4. Find the values of A and B.

$$\begin{array}{c} AB \\ +37 \\ 6A \end{array}$$

Ones column

$$B + 7 = A + 10$$

(carry = 1)

$$A = B - 3$$

Tens column

$$\begin{aligned} &A + 3 + 1 = 6 \\ &A + 4 = 6 \\ &A = 2 \end{aligned}$$

Substitute into

$$\begin{aligned} &A = B - 3 \\ &2 = B - 3 \\ &B = 5 \end{aligned}$$

Verification

$$25 + 37 = 62$$

Correct.

Answer

$$\boxed{A = 2,\ B = 5}$$

A. Assertion–Reason Questions

1.

Assertion (A): If 575a is divisible by 11, then the value of a is 3.

Reason (R): If the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or divisible by 11, then the number is divisible by 11.

Verification:

For 575a,

Odd places (from right): $a + 7$

Even places: $5 + 5 = 10$

Difference:

$$(a + 7) - 10 = a - 3$$

For divisibility by 11, difference must be 0 or a multiple of 11.

Since $a$is a digit (0–9),

$$a - 3 = 0 \Rightarrow a = 3$$

So Assertion is **True**.

Reason is the correct divisibility rule for 11, hence **True**.

Answer: (a) Both A and R are true and R is the correct explanation of A.

2.

Assertion (A): The numbers 189, 207, 225 and 261 which are divisible by 9, are also divisible by 3.

Reason (R): If a number is divisible by another number then it is divisible by each of the factors of that number.

Since

$$9 = 3 \times 3$$

Any number divisible by 9 must also be divisible by 3.

Both statements are true and R explains A.

Answer: (a)

3.

Assertion (A): If $1A \times A = 9A$, then the value of A is 5.

Verification:

$$1A = 10 + A$$

$$(10 + A) \times A = 90 + A$$

Try $A = 5$

$$15 \times 5 = 75$$

Not 95.

Try $A = 9$

$$19 \times 9 = 171$$

Not 99.

No value of Agives $9A$.

So Assertion is **False**.

Reason: "Each digit in the puzzle must be represented by just one letter" is **True**.

Answer: (c) A is false but R is true.

B. Case Study Based Questions

Given the 5-digit number:

$$A69BC$$

is divisible by 3.

Therefore,

$$A+6+9+B+C$$

must be a multiple of 3.

$$A+B+C+15$$

must be divisible by 3.

(i) If $C = 2A$and $C = 8$, find B.

$$C = 8, A = 4$$

Sum of digits:

$$4+6+9+B+8 = 27+B$$

For divisibility by 3,

$$B = 0,3,6,9$$

Answer: $\boxed{0,\ 3,\ 6,\ 9}$

(ii) If $C = 8$and $B = 3$, find A.

Sum:

$$A + 6 + 9 + 3 + 8 = A + 26$$

For divisibility by 3,

$$A + 26 \equiv 0 (\text{mod} 3)$$
$$A + 2 \equiv 0 (\text{mod} 3)$$
$$A \equiv 1 (\text{mod} 3)$$

Possible digits:

$$\boxed{1,\ 4,\ 7}$$

(iii) If $A = 4$and $B = 3$, find C.

Sum:

$$4 + 6 + 9 + 3 + C = 22 + C$$

For divisibility by 3,

$$22 + C \equiv 0 (\text{mod} 3)$$
$$1 + C \equiv 0 (\text{mod} 3)$$
$$C \equiv 2 (\text{mod} 3)$$

Possible digits:

$$\boxed{2,\ 5,\ 8}$$

C. Maths Booster

✓ = Divisible, ✗ = Not Divisible

Number	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
98760	✓	✓	✓	✓	✓	✗	✓	✓	✓	✗

Number	2	3	4	5	6	7	8	9	10	11
4315	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗
999	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗
12453	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗
54	✓	✓	✗	✗	✓	✗	✗	✓	✗	✗
210	✓	✓	✗	✓	✓	✓	✗	✗	✓	✗
5678	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗
30051	✗	✓	✗	✗	✗	✗	✗	✓	✗	✗
1325	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗
48755	✗	✗	✗	✓	✗	✗	✗	✗	✗	✗

Chapter 6: We Distribute, Yet Things Multiply

NCERT CORNER

Figure It Out

IN TEXT – 1

Q1. What would we get if we had expanded $(\boldsymbol{a}+\mathbf{1})(\boldsymbol{b}+\mathbf{1})$by first taking $(\boldsymbol{b}+\mathbf{1})$as a single term? Try it.

Using distributive law:

$$(a+1)(b+1) = a(b+1) + 1(b+1)$$

Now expand:

$$= ab + a + b + 1$$

Answer:

$$\boxed{(a+1)(b+1) = ab + a + b + 1}$$

We get the same result as before.

Q2. Will the product always increase? Find 3 examples where the product decreases.

No, the product does not always increase.

When one or both numbers are negative, the product may decrease.

Example 1

Original product:

$$2 \times (-3) = -6$$

After increasing both numbers by 1:

$$3 \times (-2) = -6$$

After increasing again:

$$4 \times (-1) = -4$$

Product decreases initially.

Example 2

$$(-5) \times 2 = -10$$

Increase both by 1:

$$(-4) \times 3 = -12$$

Product decreases from -10 to -12.

Example 3

$$(-4) \times 1 = -4$$

Increase both by 1:

$$(-3) \times 2 = -6$$

Product decreases.

Answer:

Examples:

$$(-5)(2) = -10, (-4)(3) = -12$$
$$(-4)(1) = -4, (-3)(2) = -6$$
$$(-6)(2) = -12, (-5)(3) = -15$$

Hence, the product does **not always increase**.

IN TEXT – 2

Use Identity 1:

$$(a + x)(b + y) = ab + ay + bx + xy$$

(i) One number is decreased by 2 and the other increased by 3.

New product:

$$(a - 2)(b + 3)$$

Expanding:

$$= ab + 3a - 2b - 6$$

Change in product:

$$(ab + 3a - 2b - 6) - ab$$

$$= 3a - 2b - 6$$

Answer:

$$\boxed{\text{Change in product} = 3a - 2b - 6}$$

(ii) Both numbers are decreased, one by 3 and the other by 4.

New product:

$$(a - 3)(b - 4)$$

Expanding:

$$= ab - 4a - 3b + 12$$

Change in product:

$$(ab - 4a - 3b + 12) - ab$$
$$= -4a - 3b + 12$$

Answer:

$$\boxed{\text{Change in product} = -4a - 3b + 12}$$

FIGURE IT OUT

Q. 1

(i) $(3 + u)(v - 3)$

$$= 3(v - 3) + u(v - 3)$$
$$= 3v - 9 + uv - 3u$$
$$\boxed{uv + 3v - 3u - 9}$$

(ii) $\frac{2}{3}(15 + 6a)$

$$= \frac{2}{3} \times 15 + \frac{2}{3} \times 6a$$
$$= 10 + 4a$$
$$\boxed{4a + 10}$$

(iii) $(\mathbf{10}\boldsymbol{a} + \boldsymbol{b})(\mathbf{10}\boldsymbol{c} + \boldsymbol{d})$

$$= 10a(10c + d) + b(10c + d)$$
$$= 100ac + 10ad + 10bc + bd$$
$$\boxed{100ac + 10ad + 10bc + bd}$$

(iv) $(\mathbf{3} - \boldsymbol{x})(\boldsymbol{x} - \mathbf{6})$

$$= 3(x - 6) - x(x - 6)$$
$$= 3x - 18 - x^2 + 6x$$
$$\boxed{-x^2 + 9x - 18}$$

(v) $(-\mathbf{5}\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{c} + \boldsymbol{d})$

$$= -5a(c + d) + b(c + d)$$
$$= -5ac - 5ad + bc + bd$$
$$\boxed{-5ac - 5ad + bc + bd}$$

(vi) $(\mathbf{5} + \boldsymbol{z})(\boldsymbol{y} + \mathbf{9})$

$$= 5(y + 9) + z(y + 9)$$
$$= 5y + 45 + zy + 9z$$
$$\boxed{zy + 5y + 9z + 45}$$

Question 2.
Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression pq, as shown in the figure, write the expressions for the other

numbers in the grid.

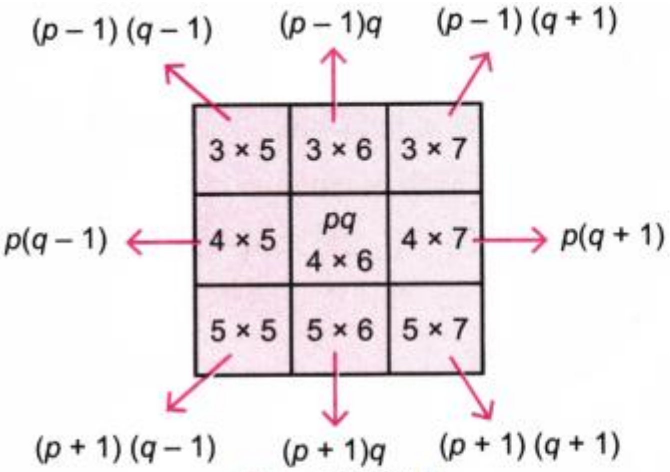
x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

	pq	

Solution:

Here,



Question 3.

Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

Solution:

Let the numbers be a and b.

Then, ab = (a + 2) (b – 4)

⇒ ab = ab – 4a + 2b – 8

⇒ ab – ab + 4a + 8 = 2b

⇒ 4a + 8 = 2b [Divide throughout by 2]

⇒ 2a + 4 = b

⇒ b = 2a + 4

For a = 1, b = 2 × 1 + 4 = 6

ab = 1 × 6 = 6

and $(a + 2)(b - 4) = 3 \times 2 = 6$
Hence, $ab = (a + 2)(b - 4)$
Let $a = 2$, then $b = 2 \times 2 + 4 = 8$
Let $a = 3$, then $6 = 2 \times 3 + 4 = 10$
Three such pairs are 1 and 6, 2 and 8, and 3 and 10.

Question 4.
Expand (i) $(a + ab - 3b^2)(4 + b)$, and (ii) $(4y + 7)(y + 11z - 3)$.
Solution:
(i) Here, $(a + ab - 3b^2)(4 + b)$
$= (4 + b)(a + ab - 3b^2)$
$= 4(a + ab - 3b^2) + b(a + ab - 3b^2)$
$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$
$= 4a + 5ab - 12b^2 + ab^2 - 3b^3$

(ii) Here, $(4y + 7)(y + 11z - 3)$
$= 4y(y + 11z - 3) + 7(y + 11z - 3)$
$= 4y2 + 44yz - 12y + 7y + 77z - 21$
$= 4y2 + 44yz - 5y + 77z - 21$

Question 5.
Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$, and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$. Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?
Solution:
(i) Here, $(a - b)(a + b)$
$= a(a + b) - b(a + b)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

(ii) Here, $(a - b)(a^2 + ab + b^2)$
$= a(a^2 + ab + b^2) - b(a^2 + ab + b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

(iii) We have, $(a - b)(a^3 + a^2b + ab^2 + b^3)$
$= a(a^3 + a^2b + ab^2 + b^3) - b(a^3 + a^2b + ab^2 + b^3)$
$= a^4 + a^3b + a^2b^2 + ab^3 - a^3b - a^2b^2 - ab^3 - b^4$
$= a^4 - b^4$
We observe the following pattern $(a - b)(a^n + a^{n-1}b + \ldots.. + b^n) = a^{n+1} - b^{n+1}$
Next identity in the pattern would be $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 - b^5$

IN TEXT –

Q. 1

Q1. Describe a general rule to multiply a number (of any number of digits) by 11 and write the product in one line.

General Rule for Multiplication by 11

To multiply a number by 11:

- Write the first digit.
- Add each pair of adjacent digits and write the sums in order.
- Write the last digit.
- Carry over whenever a sum exceeds 9.

(i) 94×11

Digits: 9, 4

Middle digit:

$$9 + 4 = 13$$

Write 3 and carry 1.

$$94 \times 11 = 1034$$

Adjusting carry:

$$\boxed{1034}$$

Check:

$$94 \times 11 = 1034$$

Oops! Let's calculate directly:

$$94 \times 11 = 94 \times (10 + 1)$$
$$= 940 + 94$$

$$\boxed{1034}$$

(ii) 495×11

$$495 \times 11$$

Adjacent sums:

$$4,\ 4 + 9 = 13,\ 9 + 5 = 14,\ 5$$

Using carries:

$$495 \times 11 = 5445$$

Verification:

$$495 \times 11 = 4950 + 495$$
$$\boxed{5445}$$

(iii) 3279×11

Adjacent sums:

$$3,\ 3 + 2 = 5,\ 2 + 7 = 9,\ 7 + 9 = 16,\ 9$$

Carry:

$$3279 \times 11 = 36069$$

Verification:

$$32790 + 3279 = 36069$$
$$\boxed{36069}$$

(iv) 4791256×11

Using the shortcut:

$$4,\ 11,\ 16,\ 10,\ 3,\ 7,\ 11,\ 6$$

After carrying:

$$\boxed{52703816}$$

Verification:

$$4791256 \times 11 = 52703816$$

Q2. Use this to multiply 3874×101in one line.

Since

$$101 = 100 + 1$$
$$3874 \times 101$$

Place two zeros between the number and itself:

$$3874\ \ 00\ \ 3874$$

Add:

$$387400 + 3874$$
$$\boxed{391274}$$

Q3. What could be a general rule to multiply a number by 101 and write the product in one line? Extend this rule for multiplication by 1001, 10001, ...

Rule for multiplication by 101

Write the number twice with two zeros in between.

Example:

$$563 \times 101 = 56863?$$

Actually,

$$563 \times 101 = 56300 + 563$$
$$\boxed{56863}$$

Rule for multiplication by 1001

Write the number twice with three zeros between.

Example:

$$432 \times 1001$$
$$432000 + 432$$
$$\boxed{432432}$$

Rule for multiplication by 10001

Write the number twice with four zeros between.

Example:

$$567 \times 10001$$
$$5670000 + 567$$
$$\boxed{5670567}$$

IN TEXT – 2

Q1. What if we write 65^2as $(30 + 35)^2$or $(52 + 13)^2$? Draw the figures and check the area that you get.

Using identity:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Method 1

$$65^2 = (30 + 35)^2$$

$$= 30^2 + 2(30)(35) + 35^2$$
$$= 900 + 2100 + 1225$$
$$= 4225$$

Method 2

$$65^2 = (52 + 13)^2$$
$$= 52^2 + 2(52)(13) + 13^2$$
$$= 2704 + 1352 + 169$$
$$= 4225$$

Answer

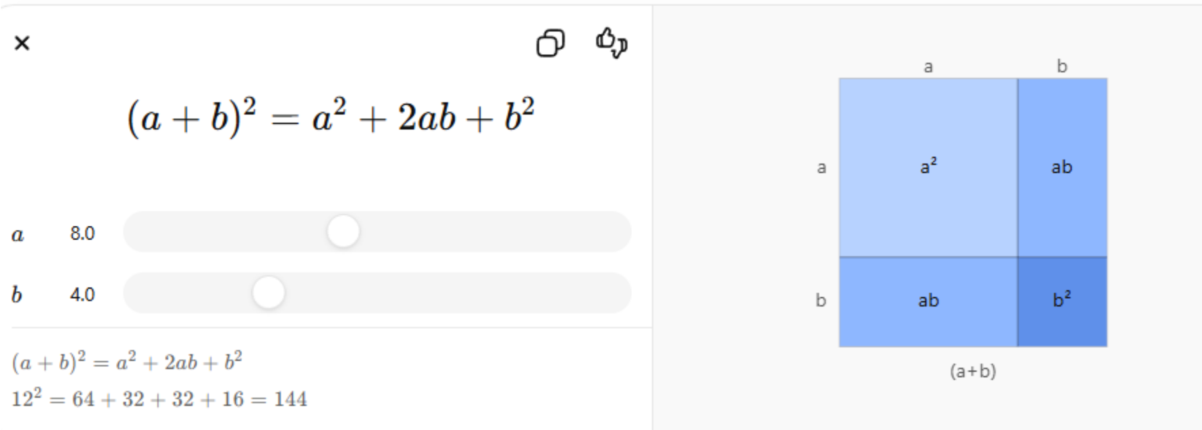
Both methods give:

$$\boxed{65^2 = 4225}$$

The area remains the same.

Q2. If $a$and $b$are any two integers, is $(a + b)^2$always greater than $a^2 + b^2$? If not, when is it greater?

Using identity:



Subtract $a^2 + b^2$from both sides:

$$(a + b)^2 - (a^2 + b^2) = 2ab$$

Therefore:

- If $ab > 0$(both have same sign), then $2ab > 0$

$$(a+b)^2 > a^2 + b^2$$

- If $ab = 0$

$$(a+b)^2 = a^2 + b^2$$

- If $ab < 0$(opposite signs)

$$(a+b)^2 < a^2 + b^2$$

Answer

$(a+b)^2$is not always greater than $a^2 + b^2$.

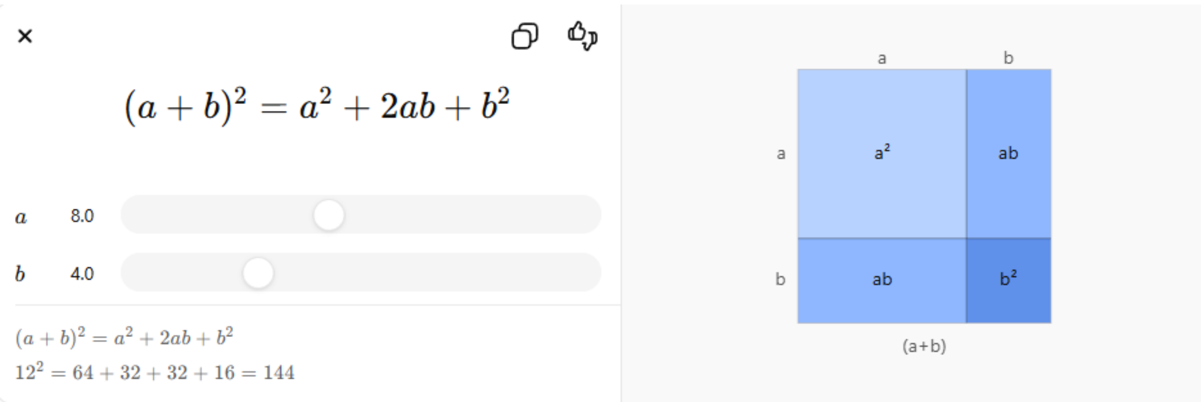
It is greater only when:

$$\boxed{ab > 0}$$

(that is, when $a$and $b$are both positive or both negative).

3. Use Identity 1A to find the values of $104^2$and 37^2

Identity 1A:



(i) 104^2

$$104 = 100 + 4$$

Using Identity:

$$
\begin{aligned}
&(100+4)^2 \\
&= 100^2 + 2(100)(4) + 4^2 \\
&= 10000 + 800 + 16 \\
&= 10816
\end{aligned}
$$

Answer:

$$\boxed{104^2 = 10816}$$

(ii) 37^2

$$
\begin{aligned}
&37 = 30 + 7 \\
&(30+7)^2 \\
&= 30^2 + 2(30)(7) + 7^2 \\
&= 900 + 420 + 49 \\
&= 1369
\end{aligned}
$$

Answer:

$$\boxed{37^2 = 1369}$$

---

4. Use Identity 1A to write the expressions

(i) $(m+3)^2$

$$
\begin{aligned}
&= (m)^2 + 2(m)(3) + 3^2 \\
&= \boxed{m^2 + 6m + 9}
\end{aligned}
$$

(ii) $(6+p)^2$

$$
\begin{aligned}
&= 6^2 + 2(6)(p) + p^2 \\
&= 36 + 12p + p^2
\end{aligned}
$$

$$= \boxed{p^2 + 12p + 36}$$

---

5. Expand $(3j + 2k)^2$

Method 1: Using Identity

$$(a + b)^2 = a^2 + 2ab + b^2$$

Here,

$$a = 3j, b = 2k$$
$$(3j + 2k)^2$$
$$= (3j)^2 + 2(3j)(2k) + (2k)^2$$
$$= 9j^2 + 12jk + 4k^2$$
$$\boxed{9j^2 + 12jk + 4k^2}$$

Method 2: Using Distributive Property

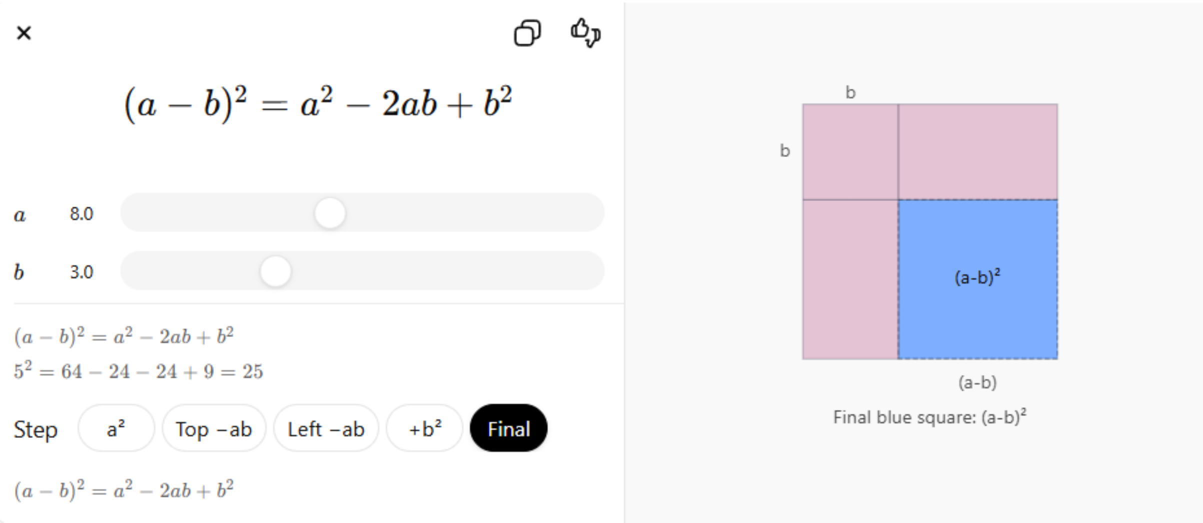
$$(3j + 2k)(3j + 2k)$$
$$= 3j(3j) + 3j(2k) + 2k(3j) + 2k(2k)$$
$$= 9j^2 + 6jk + 6jk + 4k^2$$
$$= 9j^2 + 12jk + 4k^2$$
$$\boxed{9j^2 + 12jk + 4k^2}$$

IN TEXT

1. Find the general expansion of $(a - b)^2$

$$(a - b)^2 = (a - b)(a - b)$$
$$= a(a - b) - b(a - b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

Therefore,



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2. Use $(a-b)^2$ to find

(i) 99^2

$$99 = (100-1)$$
$$99^2 = (100-1)^2$$
$$= 100^2 - 2(100)(1) + 1^2$$
$$= 10000 - 200 + 1$$
$$= 9801$$
$$\boxed{99^2 = 9801}$$

(ii) 58^2

$$58 = (60-2)$$
$$58^2 = (60-2)^2$$
$$= 60^2 - 2(60)(2) + 2^2$$
$$= 3600 - 240 + 4$$
$$= 3364$$
$$\boxed{58^2 = 3364}$$

---

3. Expand using Identity 1B and distributive property

(i) $(b-6)^2$

Using Identity:

$$= b^2 - 2(b)(6) + 6^2$$
$$= \boxed{b^2 - 12b + 36}$$

(ii) $(-2a + 3)^2$

$$= (-2a)^2 + 2(-2a)(3) + 3^2$$
$$= 4a^2 - 12a + 9$$
$$\boxed{4a^2 - 12a + 9}$$

(iii) $\left(7y - \frac{3}{4z}\right)^2$

Using

$$(a - b)^2 = a^2 - 2ab + b^2$$
$$= (7y)^2 - 2(7y)\left(\frac{3}{4z}\right) + \left(\frac{3}{4z}\right)^2$$
$$= 49y^2 - \frac{42y}{4z} + \frac{9}{16z^2}$$
$$= 49y^2 - \frac{21y}{2z} + \frac{9}{16z^2}$$
$$\boxed{49y^2 - \frac{21y}{2z} + \frac{9}{16z^2}}$$

IN TEXT

1. Take a pair of natural numbers. Calculate the sum of their squares. Can you write twice this sum as a sum of two squares?

Take the natural numbers 3 and 4.

$$3^2 + 4^2 = 9 + 16 = 25$$

Twice this sum:

$$2(25) = 50$$

Now write 50 as a sum of two squares:

$$50 = 25 + 25$$
$$50 = 5^2 + 5^2$$

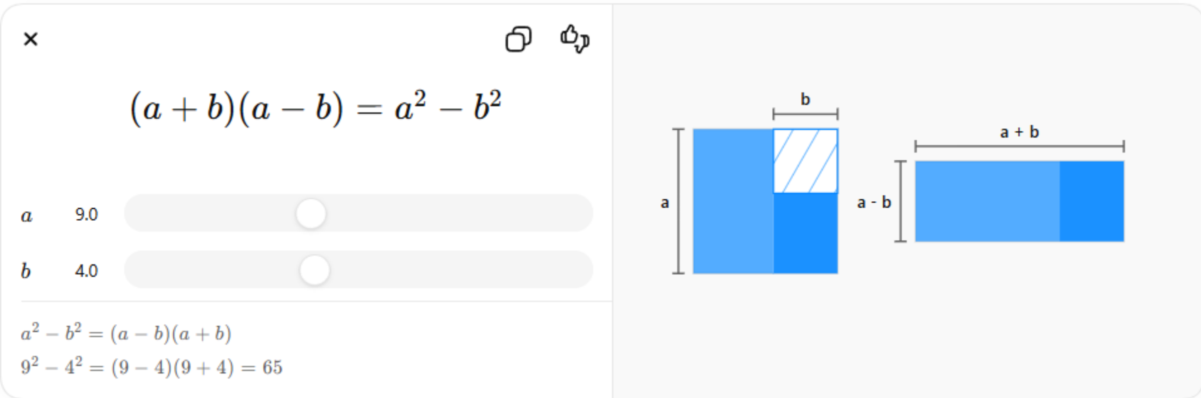
Hence, twice the sum of squares can be written as a sum of two squares.

Answer: Yes.

---

2. Use Identity 1C to calculate $98 \times 102$and 45×55

Identity 1C:



(i) 98×102

$$98 = (100 - 2)$$
$$102 = (100 + 2)$$
$$98 \times 102 = (100 - 2)(100 + 2)$$
$$= 100^2 - 2^2$$
$$= 10000 - 4$$
$$= 9996$$

Answer:

$$\boxed{9996}$$

(ii) 45×55

$$45 = (50 - 5)$$
$$55 = (50 + 5)$$

$$
\begin{aligned}
&= (50-5)(50+5) \\
&= 50^2 - 5^2 \\
&= 2500 - 25 \\
&= 2475
\end{aligned}
$$

Answer:

$$\boxed{2475}$$

---

3. Show that $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b}) = \boldsymbol{a}^2 - \boldsymbol{b}^2$

$$
\begin{aligned}
&(a+b)(a-b) \\
&= a(a-b) + b(a-b) \\
&= a^2 - ab + ab - b^2 \\
&= a^2 - b^2
\end{aligned}
$$

Hence proved.

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

MIND THE MISTAKE, MEND THE MISTAKE

(a)

Given:

$$-3p(-5p+2q)$$

Correct expansion:

$$
\begin{aligned}
&= (-3p)(-5p) + (-3p)(2q) \\
&= 15p^2 - 6pq
\end{aligned}
$$

Correct Answer:

$$\boxed{15p^2 - 6pq}$$

(b)

$$2(x-1)+3(x+4)$$
$$=2x-2+3x+12$$
$$=5x+10$$

Correct Answer:

$$\boxed{5x+10}$$

(c)

$$y+2(y+2)$$
$$=y+2y+4$$
$$=3y+4$$

Correct Answer:

$$\boxed{3y+4}$$

(d)

$$(5m+6n)^2$$
$$=(5m)^2+2(5m)(6n)+(6n)^2$$
$$=25m^2+60mn+36n^2$$

Correct Answer:

$$\boxed{25m^2+60mn+36n^2}$$

(e)

$$(-q+2)^2$$
$$=(-q)^2+2(-q)(2)+2^2$$

$$= q^2 - 4q + 4$$

Correct Answer:

$$\boxed{q^2 - 4q + 4}$$

(This one was already correct.)

(f)

$$3a(2b \times 3c)$$
$$= 3a(6bc)$$
$$= 18abc$$

Correct Answer:

$$\boxed{18abc}$$

(g)

$$\frac{1}{2}(10s - 6) + 3$$
$$= 5s - 3 + 3$$
$$= 5s$$

Correct Answer:

$$\boxed{5s}$$

(This one was already correct.)

(h)

$$5w^2 + 6w$$

These are unlike terms and cannot be added.

Correct Answer:

$$\boxed{5w^2 + 6w}$$

(i)

$$2a^3 + 3a^3 + 6a^2b + 6ab^2$$

Combine only like terms:

$$= 5a^3 + 6a^2b + 6ab^2$$

Correct Answer:

$$\boxed{5a^3 + 6a^2b + 6ab^2}$$

(j)

$$\begin{aligned} &(x+2)(x+5) \\ &= x(x+5) + 2(x+5) \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10 \end{aligned}$$

Correct Answer:

$$\boxed{x^2 + 7x + 10}$$

(This one was already correct.)

(k)

$$\begin{aligned} &(a+2)(b+4) \\ &= ab + 4a + 2b + 8 \end{aligned}$$

Correct Answer:

$$\boxed{ab + 4a + 2b + 8}$$

(l)

$$ab^2 + a^2b + a^2b^2$$

Common factor:

$$= ab(b + a + ab)$$

Correct Answer:

$$\boxed{ab(a + b + ab)}$$

Figure It Out

Question 1.
Which is greater: $(a – b)^2$ or $(b – a)^2$? Justify your answer.
Solution:
Here, $(a – b)^2 = a^2 + b^2 – 2ab$ ……….(1)
and $(b – a)^2 = b^2 + a^2 – 2ba$
$b^2 + a^2 = a^2 + b^2$ and ba = ab
$(b – a)^2 = a^2 + b^2 – 2ab$ ……….(2)
Comparing (1) and (2), we get $(a – b)^2 = (b – a)^2$

Question 2.
Express 100 as the difference of two squares.
Solution:
$a^2 – b^2 = 100$
(a + b) (a – b) = 100
[100 = 1 × 100, 2 × 50, 4 × 25, 5 × 20, 10 × 10]
We can take anyone
Let us take 50 × 2 = 100
Hence, (a + b) (a – b) = 50 × 2
a + b = 50 ………(1)
a – b = 2 …….(2)
Adding (1) and (2)
2a = 52
⇒ a = 26

Substituting a = 26 in (1)
$26 + b = 50$
$\Rightarrow b = 50 - 26 = 24$
Let us check $26^2 - 24^2 = 676 - 576 = 100$
Hence $26^2 - 24^2 = 100$

Question 3.
Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learned so far.
Solution:
(i) $406^2 = (400 + 6)^2$
$= 400^2 + 2 \times 400 \times 6 + 6^2$
$= 160000 + 4800 + 36$
$= 164836$

(ii) $72^2 = (50 + 22)^2$
$= 50^2 + 2 \times 50 \times 22 + 22^2$
$= 2500 + 2200 + 484$
$= 5184$

(iii) $145^2 = (150 - 5)^2$
$= 150^2 - 2 \times 150 \times 5 + 5^2$
$= 22500 - 1500 + 25$
$= 21025$

(iv) $1097^2 = (1100 - 3)^2$
$= 1100^2 - 2 \times 1100 \times 3 + 3^2$
$= 1210000 - 6600 + 9$
$= 1203409$

(v) $124^2 = (100 + 24)^2$
$= 100^2 + 2 \times 100 \times 24 + 24^2$
$= 10000 + 4800 + 576$
$= 15376$

Question 4.
Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.
Solution:
Pattern 1
$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$
Case-I
Let $a = 4$, $b = 2$

LHS = $2(4^2 + 2^2) = 2 \times (16 + 4) = 40$
RHS = $(4 + 2)^2 + (4 - 2)^2 = 36 + 4 = 40$
∴ Pattern 1 holds for counting numbers.

Case-II
Let a = -4, b = -2
LHS = $2((-4)^2 + (-2)^2)$
$= 2 \times (16 + 4)$
$= 2 \times 20$
$= 40$
RHS = $(-4 + (-2))^2 + (-4 - (-2))^2$
$= (-4 - 2)^2 + (-4 + 2)^2$
$= (-6)^2 + (-2)^2$
$= 36 + 4$
$= 40$
LHS = RHS
∴ Pattern 1 holds for negative integers also.

Case-III
Let $a = \frac{1}{2}$, $b = \frac{1}{3}$
LHS $= 2\left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2\right)$
$= 2\left[\frac{1}{4} + \frac{1}{9}\right] = 2 \times \frac{13}{36} = \frac{13}{18}$
RHS $= \left(\frac{1}{2} + \frac{1}{3}\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\right)^2$
$= \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2 = \frac{25}{36} + \frac{1}{36}$
$= \frac{26}{36} = \frac{13}{18}$
The pattern holds for fractions also.

Pattern 2
$a^2 - b^2 = (a + b)(a - b)$
Case-I
Let a = 5, b = 3
LHS = $5^2 - 3^2 = 25 - 9 = 16$
RHS = $(5 + 3)(5 - 3) = 8 \times 2 = 16$

∴ LHS = RHS
∴ Pattern 2 holds for counting numbers.

Case-II
Let a = -5, b = -3
Now, LHS = $(-5)^2 - (-3)^2 = 25 - 9 = 16$
and RHS = [(-5) + (-3)] [(-5) – (-3)]
= (-5 – 3) (-5 + 3)
= (-8) (-2)
= 16
∴ LHS = RHS
∴ Pattern 2 holds for negative integers also.

Case-III
Let a = -5, b = -3
Now, LHS = $(-5)^2 - (-3)^2 = 25 - 9 = 16$
and RHS = [(-5) + (-3)] [(-5) – (-3)]
= (-5 – 3) (-5 + 3)
= (-8) (-2)
= 16
∴ LHS = RHS
∴ Pattern 2 holds for negative integers also.

IN TEXT – Q1

Verify that $x^2 - xy$, $x(x - y)$ and $x(x + 2y) - 3xy$ are equivalent.

Expression 1

$$x^2 - xy$$

Expression 2

$$x(x - y)$$
$$= x^2 - xy$$

Expression 3

$$x(x + 2y) - 3xy$$

$$= x^2 + 2xy - 3xy$$
$$= x^2 - xy$$

All three expressions simplify to:

$$\boxed{x^2 - xy}$$

Hence they are equivalent.

For $x = 8$, $y = 3$

$$x^2 - xy$$
$$= 8^2 - (8)(3)$$
$$= 64 - 24$$
$$= 40$$

Area of shaded region

$$\boxed{\text{40 square units}}$$

IN TEXT – Q2

Given

$$p = 6, r = 3.5, s = 9$$

The dashed region is the inner rectangle.

Length

$$s - r$$

Breadth

$$p - r$$

Area

$$= (s - r)(p - r)$$

Substituting values:

$$= (9 - 3.5)(6 - 3.5)$$
$$= 5.5 \times 2.5$$
$$= 13.75$$
$$\boxed{13.75 \text{ sq units}}$$

TILE PATTERN

(i) Number of tiles

Step 1:

$$3 \times 3 = 9$$

Step 2:

Outer 4×4square with inner 2×2removed

$$16 - 4 = 12$$

Step 3:

Outer 5×5square with inner 3×3removed

$$25 - 9 = 16$$

Answers:

$$\boxed{9,\ 12,\ 16}$$

(ii) Step 4

Outer 6×6

Inner 4×4

$$36 - 16 = 20$$

$$\boxed{20}$$

Step 10

Outer 12×12

Inner 10×10

$$144 - 100$$
$$= 44$$
$$\boxed{44}$$

(iii) Algebraic expression

For Step n:

Outer square:

$$(n+2)^2$$

Inner square:

$$n^2$$

Tiles

$$(n+2)^2 - n^2$$
$$= n^2 + 4n + 4 - n^2$$
$$= 4n + 4$$
$$\boxed{4n+4}$$

IN TEXT – Q3

Verify

$$(m+n)^2 - 4mn = (n-m)^2$$

LHS:

$$
\begin{aligned}
&(m+n)^2 - 4mn \\
&= m^2 + 2mn + n^2 - 4mn \\
&= m^2 - 2mn + n^2 \\
&= (m-n)^2 \\
&= (n-m)^2
\end{aligned}
$$

Hence verified.

$$\boxed{(m+n)^2 - 4mn = (n-m)^2}$$

Figure It Out

Question 1.
Compute these products using the suggested identity.
(i) 46^2 using Identity 1A for $(a + b)^2$
(ii) 397×403 using Identity 1C for $(a + b)(a – b)$
(iii) 91^2 using Identity 1B for $(a – b)^2$
(iv) 43×45 using Identity 1C for $(a + b)(a – b)$
Solution:
(i) $46^2 = (40 + 6)^2$
$= 40^2 + 2 \times 40 \times 6 + 6^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]
$= 1600 + 480 + 36$
$= 2116$

(ii) $397 \times 403 = (400 – 3)(400 + 3)$ [$\because (a + b) \times (a – b) = a^2 – b^2$]
$= 400^2 – 3^2$
$= 160000 – 9$
$= 159991$

(iii) $91^2 = (100 – 9)^2$
$= 100^2 – 2 \times 100 \times 9 + 9^2$ [$\because (a – b)^2 = a^2 + b^2 – 2ab$]
$= 10000 – 1800 + 81$
$= 8281$

(iv) $43 \times 45 = (44 – 1)(44 + 1)$ [$\because a^2 – b^2 = (a + b) \times (a – b)$]
$= 44^2 – 1^2$
$= 1936 – 1$
$= 1935$

Question 2.
Use either a suitable identity or the distributive property to find each of the following products.
(i) $(p - 1)(p + 11)$
(ii) $(3a - 9b)(3a + 9b)$
(iii) $-(2y + 5)(3y + 4)$
(iv) $(6x + 5y)^2$
(v) $(2x - \frac{1}{2})^2$
(vi) $(7p) \times (3r) \times (p + 2)$
Solution:
(i) $(p - 1)(p + 11)$
$= p(p + 11) - 1(p + 11)$
$= p^2 + 11p - p - 11$
$= p^2 + 10p - 11$

(ii) $(3a - 9b)(3a + 9b)$
$= (3a)^2 - (9b)^2$
$= 9a^2 - 81b^2$

(iii) $-(2y + 5)(3y + 4)$
$= (-2y - 5)(3y + 4)$
$= -2y(3y + 4) - 5(3y + 4)$
$= -6y^2 - 8y - 15y - 20$
$= -6y^2 - 23y - 20$

(iv) $(6x + 5y)^2$
$= (6x)^2 + 2(6x)(5y) + (5y)^2$
$= 36x^2 + 60xy + 25y^2$

(v) $(2x - \frac{1}{2})2$
$= (2x)^2 - 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$
$= 4x^2 - 2x + \frac{1}{4}$

(vi) $(7p) \times (3r) \times (p + 2)$
$= 7p \times 3r \times (p + 2)$
$= 21pr(p + 2)$
$= 21pr \times p + 21pr \times 2$
$= 21p^2r + 42pr$

Question 3.
For each statement, identify the appropriate algebraic expression(s).
(i) Two more than a square number.

- $2 + s$
- $(s + 2)^2$
- $s^2 + 2$
- $s^2 + 4$
- $2s^2$
- 2^2s

(ii) The sum of the squares of two consecutive numbers

- $m^2 + n^2$
- $(m + n)^2$
- $m^2 + 1$
- $m^2 + (m + 1)^2$
- $m^2 + (m - 1)^2$
- $(m + (m + 1))^2$
- $(2m)^2 + (2m + 1)^2$

Solution:
(i) Two more than a square is $s^2 + 2$
(ii) Sum of the squares of two consecutive numbers is $m^2 + (m + 1)^2$

Question 4.
Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

February						
Su	**M**	**Tu**	**W**	**Th**	**F**	**Sa**
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Find products of numbers lying along each diagonal – $4 \times 12 = 48$, $5 \times 11 = 55$.
Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.
Hint: Label the numbers in each 2 by 2 square as

a	$(a+1)$
$a+7$	$(a+8)$

Solution:

Case-I

6	7
13	14

Here, 6 × 14 = 84

13 × 7 = 91

Difference = 91 – 84 = 7

Case-II

9	10
16	17

Here, 9 × 17 = 153

16 × 10 = 160

Difference = 160 – 153 = 7

We observe that the difference of the diagonal products in both cases is always 7.

Question 5.

Verify which of the following statements are true.

(i) (k + 1) (k + 2) – (k + 3) is always a multiple of 2.

(ii) (2q + 1) (2q – 3) is a multiple of 4.

(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.

(iv) $(6n + 2)^2 - (4n + 3)^2$ is 5 less than a square number.

Solution:

(i) (k + 1)(k + 2) – (k + 3) is always a multiple of 2

Let k = 5, Then (5 + 1) (5 + 2) – (5 + 3)

= 6 × 7 – 8

= 42 – 8

= 34

34 is a multiple of 2.

∴ The statement is true.

(ii) (2q + 1) (2q – 3) is a multiple of 4.

Let q = 3, Then (6 + 1) (6 – 3)

$= 7 \times 3$
$= 21$
21 is not a multiple of 4
$\therefore$ The statement is false.

(iii) The square of an even number is a multiple of 4.
$2^2 = 4 = 4 \times 1$
$4^2 = 16 = 4 \times 4$
$6^2 = 36 = 4 \times 9$
$\therefore$ The statement is true.
The square of an odd number is 1 more than a multiple of 8.
$3^2 = 9 = 8 \times 1 + 1$
$5^2 = 25 = 8 \times 3 + 1$
$7^2 = 49 = 8 \times 6 + 1$
$\therefore$ The statement is true.

(iv) $(6n + 2)^2 – (4n + 3)^2$ is 5 less than a square number.
Let n = 2, $(6 \times 2 + 2)^2 – (4 \times 2 + 3)^2$
$= 14^2 – 11^2$
$= 196 – 121$
$= 75$
$= 80 – 5$
But 80 is not a square number.
$\therefore$ The statement is false.

Question 6.
A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?
Solution:
Let the numbers be x and y.
$x = 7a + 3$, $y = 7b + 5$
Sum $= x + y$
$= 7a + 3 + 7b + 5$
$= 7(a + b) + 8$
$= 7(a + b) + 7 + 1$
$= 7(a + b + 1) + 1$
$\therefore$ The remainder on division by 7 is 1.
Difference $= x – y$
$= (7a + 3) – (7b + 5)$
$= 7a + 3 – 7b – 5$

= 7(a – b) – 2
= 7(a – b) – 1 + 5 (∵ -2 = -7 + 5)
= 7(a – b – 1) + 5
∴ The remainder on division by 7 is 5.
Product = xy
= (7a + 3) (7b + 5)
= 49ab + 35a + 21b + 15
= (49ab + 35a + 21b + 14) + 1
= 7(7ab + 5a + 3b + 2) + 1
∴ The remainder on division by 7 is 1.

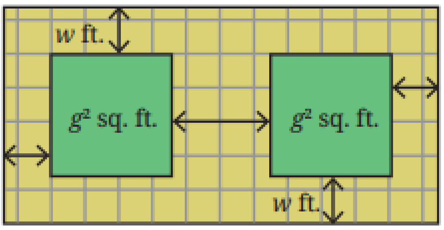
Question 7.
Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.
Solution:
Let us take the numbers 7, 8, 9
Now, $8^2 – 7 \times 9 = 64 – 63 = 1$
Let us take the numbers 10, 11, 12
Then $11^2 – 10 \times 12 = 121 – 120 = 1$
Generalizing:
Let the numbers be a – 1, a, a + 1
Then $a^2 – (a + 1) (a – 1) = 1$
LHS = $a^2 – (a + 1)(a – 1)$
= $a^2 – (a^2 – 1)$
= $a^2 – a^2 + 1$
= 1
∴ LHS = RHS
Hence, the identity is correct.

Question 8.
What is the algebraic expression describing the following steps: add any two numbers? Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.
Solution:
Let the two numbers be a and b.
Step 1: a + b
Step 2: (a + b) × \frac {1}{2} (a + b)
∴ (a + b) × \frac {1}{2} (a + b) = \frac {1}{2} $(a + b)^2$

Question 9.
Which is larger? Find out without fully computing the product.
(i) 14 × 26 or 16 × 24
(ii) 25 × 75 or 26 × 74
Solution:
(i) Let p = 14 × 26
p' = 16 × 24
= (14 + 2) (26 – 2)
= 14 × 26 + 2 × 26 – 14 × 2 – 2 × 2
= 14 × 26 + 2(26 – 14 – 2)
= 14 × 26 + 2 × 10
p' = p + 2 × 10
∴ p' > p or 16 × 24 > 14 × 26

(ii) Let p = 25 × 75
p' = 26 × 74
= (25 + 1) (75 – 1)
= 25 × 75 + 75 × 1 – 25 × 1 – 1 × 1
= p + (75 – 25 – 1)
= p + 49
∴ p' > p or 26 × 74 > 25 × 75

Question 10.
A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to he tiled. Write an expression for the area that needs to be tiled.

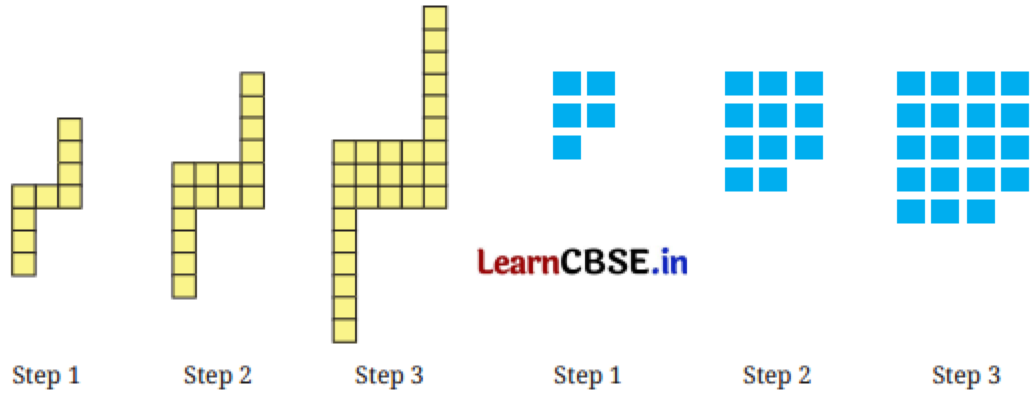


Solution:
Length = w + g + 2w + g + w = 4w + 2g
Breadth = w + g + w = 2w + g
Area of park = (4w + 2g) (2w + g)
= $8w^2 + 4wg + 4wg + 2g^2$
= $8w^2 + 8wg + 2g^2$
Area of path = Area of park – Area of green cover
= $8w^2 + 8wg + 2g^2 – 2g^2$

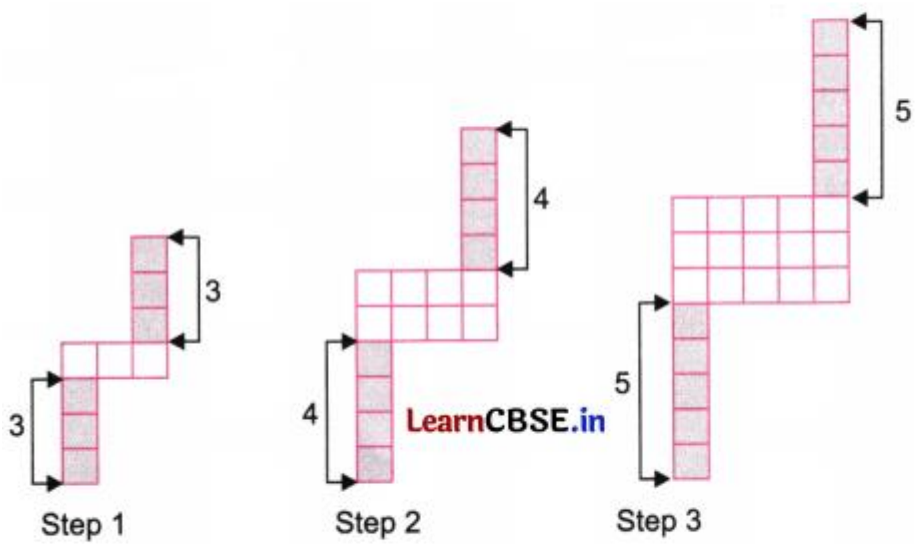
$= 8w^2 + 8wg$

$\therefore (8w^2 + 8wg)$ sq. feet area needs to be tiled.

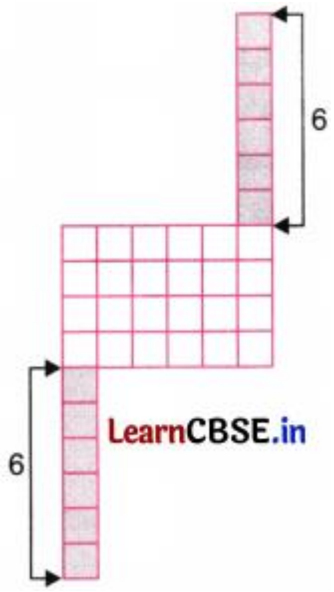
Question 11.
For each pattern shown below,
(i) Draw the next figure in the sequence.
(ii) How many basic units are there in Step 10?
(iii) Write an expression to describe the number of basic units in Step y.



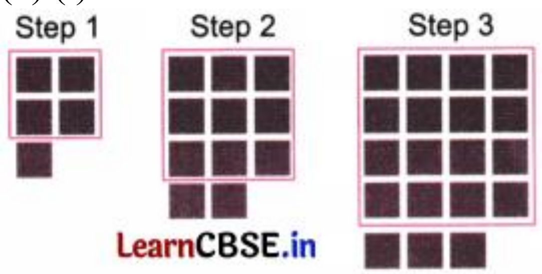
Solution:
(a)



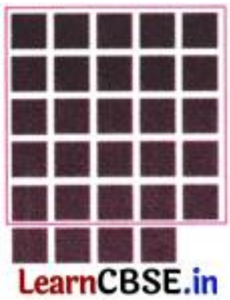
Step 1: 2 vertical strips of 3 units each + 1 vertical strip of 3 units
= 3 strips of 3 units each
= 9 units squares
$= (1 + 2)^2$ unit squares
Step 2: 4 strips of 4 units each
= 16 units squares
$= (2 + 2)^2$ unit squares
Step 3: 5 strips of 5 units each
= 25 units squares
$= (3 + 2)^2$ unit squares



Step 4: (i) 6 strips of 6 units each = 2 are vertical and 4 are horizontal
(ii) Number of unit squares in step 10 = $(10 + 2)^2 = 144$
(iii) Number of unit squares in step y = $(y + 2)^2$

(b) (i)


Number of unit squares in step 1 = 5 = $2^2 + 1$
Number of unit squares in step 2 = 11 = $3^2 + 2$
Number of unit squares in step 3 = 19 = $4^2 + 3$



(ii) Step 1 has $(1 + 1)^2 + 1$ or 5 squares
Step 2 has $(2 + 1)^2 + 2$ or 11 squares
Step 3 has $(3 + 1)^2 + 3$ or 19 squares
Hence step 10 has $(10 + 1)^2 + 10$ or 131 squares
(iii) Step y has $[(y + 1)^2 + y]$ squares

EXAM TIME

A. Multiple Choice Questions

1.

$$(-25)(6+4) = (-25)(10) = -250$$

Also,

$$(-25) \times 6 + (-25) \times 4 = -150 - 100 = -250$$

But

$$(-25) \times 6 \times 4 = -600$$

Answer: (c)

2.

$$\begin{aligned} &(-9)(-3) + (-9)(4) \\ &= 27 - 36 \\ &= -9 \end{aligned}$$

Answer: (b)

3.

$$18(12 - 3)$$

Using distributive property:

$$= 18 \times 12 - 18 \times 3$$

Answer: (c)

4.

$$\begin{aligned} &(5 + a)(5 - b) \\ &= 25 - 5b + 5a - ab \end{aligned}$$

$$= 5a - 5b - ab + 25$$

Answer: (d)

5.

$$\frac{3}{4}a(a + 4b - 12)$$
$$= \frac{3}{4}a^2 + 3ab - 9a$$

Answer: (b)

6.

Original product:

$$ab$$

New product:

$$a(b + 1) = ab + a$$

Increase

$$= (ab + a) - ab = a$$

Answer: (c)

7.

Increase in product:

$$(37)(43) - (36)(42)$$
$$1591 - 1512$$
$$= 79$$

Answer: (b)

8.

$$38 \times 42 - 37 \times 43$$
$$1596 - 1591$$
$$= 5$$

Answer: (c)

9.

$$42 \times 56 = 2352$$

New product:

$$48 \times 64 = 3072$$

Increase:

$$3072 - 2352 = 720$$

Answer: (a)

10.

$$(a + 4)^2$$
$$= a^2 + 8a + 16$$

Answer: (b)

11.

$$104^2 = (100 + 4)^2$$
$$= 10000 + 800 + 16$$
$$= 10816$$

Answer: (d)

12.

$$(x-16)^2$$
$$= x^2 - 32x + 256$$

Answer: (c)

13.

$$(a+b)^2 + (a-b)^2$$
$$= 729 + 81$$
$$= 810$$
$$2(a^2 + b^2) = 810$$
$$a^2 + b^2 = 405$$

Answer: (b)

14.

$$u + v = 13, u - v = 1$$

Adding:

$$2u = 14$$
$$u = 7$$
$$v = 6$$
$$uv = 42$$

Answer: (d)

15.

$$47 \times 53 = (50-3)(50+3)$$

Answer: (c)

16.

$$\begin{aligned}&537 \times 11\\&= 5370 + 537\\&= 5907\end{aligned}$$

Answer: (a)

17.

$$\begin{aligned}&326 \times 101\\&= 32600 + 326\\&= 32926\end{aligned}$$

Answer: (b)

18.

$$\begin{aligned}&52 \times 1001\\&= 52000 + 52\\&= 52052\end{aligned}$$

Answer: (d)

19.

$$\begin{aligned}&462 \times 99\\&= 462(100 - 1)\\&= 46200 - 462\\&= 45738\end{aligned}$$

Answer: (c)

20.

$$632 \times 999$$
$$= 632(1000 - 1)$$
$$= 632000 - 632$$
$$= 631368$$

Answer: (a)

B. Fill in the Blanks

1.

$$\frac{4}{3}(3x + 8)$$
$$= 4x + \frac{32}{3}$$

Answer: $\boxed{4x + \frac{32}{3}}$

2.

$$(6 + x)(8 + x)$$
$$= x^2 + 14x + 48$$

Answer: $\boxed{x^2 + 14x + 48}$

3.

Increase in $ab$when both increase by 1:

$$(a + 1)(b + 1) - ab$$
$$= ab + a + b + 1 - ab$$
$$= a + b + 1$$

Answer: $\boxed{a + b + 1}$

4.

$$(a+1)(b-1)-ab$$
$$= ab-a+b-1-ab$$
$$= b-a-1$$

Answer: $\boxed{b-a-1}$

5.

$$21\times 25-16\times 18$$
$$= 525-288$$
$$= 237$$

Answer: $\boxed{237}$

6.

$$(7+7x)^2$$
$$= 49+98x+49x^2$$

Answer: $\boxed{49x^2+98x+49}$

7.

$$(2m+n)(2m-n)$$
$$= 4m^2-n^2$$

Answer: $\boxed{4m^2-n^2}$

8.

$$206^2=(200+6)^2$$
$$= 40000+2400+36$$
$$= 42436$$

Answer: $\boxed{42436}$

9.

$$56 \times 101$$
$$= 5656$$

Answer: $\boxed{5656}$

10.

$$732 \times 99$$
$$= 732(100 - 1)$$
$$= 73200 - 732$$
$$= 72468$$

Answer: $\boxed{72468}$

C. True / False

1. $a(b + c) = ab + ac$, not $ab + c$→ **False**
2. Increase should be $mb + an + mn$, not $ma + mn + ab$→ **False**
3. $(3x - 4y)^2 = 9x^2 - 24xy + 16y^2$→ **True**
4. $(a + 16)(a - 16) = a^2 - 256$→ **False**
5. $103 \times 97 = (100 + 3)(100 - 3) = 9991$→ **True**
6. $(4n + 3)^2 - (4n - 3)^2 = 48n$→ **False**
7. $125 \times 11 = 1375$→ **True**
8. $7345 \times 101 = 741845$→ **True**
9. $314 \times 1001 = 314314$→ **True**
10. $123 \times 999 = 122877$→ **True**

D. Match the Columns

Column I	**Column II**
(a) $(a+2)^2$	(iii) a^2+4a+4
(b) $(3-b)^2$	(i) $9-6b+b^2$
(c) $(2+x)(3+y)$	(iv) $xy+3x+2y+6$
(d) $3\left(\frac{3}{2}x+6\right)$	(ii) $\frac{9}{2}x+18$

Answer:

$$\boxed{(a) \rightarrow (iii),\ (b) \rightarrow (i),\ (c) \rightarrow (iv),\ (d) \rightarrow (ii)}$$

E. Very Short Answer Type Questions

1. Find the value of $\frac{4}{5}(\mathbf{15x+20y})$

$$= \frac{4}{5}(15x) + \frac{4}{5}(20y)$$
$$= 12x + 16y$$

Answer: $\boxed{12x+16y}$

---

2. Find the increase in $\mathbf{21 \times 42}$when $\mathbf{21}$ is increased by 1.

Original product:

$$21 \times 42 = 882$$

New product:

$$22 \times 42 = 924$$

Increase:

$$924 - 882 = 42$$

Answer: $\boxed{42}$

---

3. Find the product of 56×101

$$56(100 + 1)$$
$$5600 + 56$$
$$= 5656$$

Answer: $\boxed{5656}$

---

4. Find the area of a square with side $y + 4$

$$\text{Area} = (y + 4)^2$$
$$= y^2 + 8y + 16$$

Answer: $\boxed{y^2 + 8y + 16}$

---

5. Find the value of 97^2

$$97^2 = (100 - 3)^2$$
$$= 10000 - 600 + 9$$
$$= 9409$$

Answer: $\boxed{9409}$

---

6. Find the value of $(2a + b)^2 + (2a - b)^2$

$$(2a + b)^2 = 4a^2 + 4ab + b^2$$
$$(2a - b)^2 = 4a^2 - 4ab + b^2$$

Adding:

$$8a^2 + 2b^2$$

Answer: $\boxed{8a^2 + 2b^2}$

---

7. What is the value of 13×99?

$$13(100-1)$$
$$1300-13$$
$$=1287$$

Answer: $\boxed{1287}$

F. Short Answer Type Questions

1. In the product 576×321, if both numbers are increased by 1, find the increase.

Increase:

$$(a+1)(b+1)-ab$$
$$=a+b+1$$

Here,

$$a=576, b=321$$
$$576+321+1=898$$

Answer: $\boxed{898}$

---

2. What is the sum of $46 \times 101$and 12×10001?

$$46 \times 101=4646$$
$$12 \times 10001=120012$$

Sum:

$$120012+4646=124658$$

Answer: $\boxed{124658}$

---

3. If $p+q=16$and $p-q=4$, find p^2+q^2.

Using identity:

$$(p+q)^2 + (p-q)^2 = 2(p^2+q^2)$$
$$16^2 + 4^2 = 2(p^2+q^2)$$
$$256 + 16 = 272$$
$$2(p^2+q^2) = 272$$
$$p^2 + q^2 = 136$$

Answer: $\boxed{136}$

---

4. Find the shaded area.

Outer square side:

$$3p + 1$$

Outer area:

$$(3p+1)^2$$

Inner square side:

$$(3p+1) - 2p = p + 1$$

Inner area:

$$(p+1)^2$$

Shaded area:

$$(3p+1)^2 - (p+1)^2$$
$$= 9p^2 + 6p + 1 - (p^2 + 2p + 1)$$
$$= 8p^2 + 4p$$

Answer: $\boxed{8p^2 + 4p}$

---

5. Number of circles in Step 14

Observe:

Step 1 = 3 circles

Step 2 = 8 circles

Step 3 = 15 circles

Pattern:

$$\begin{gathered}3 = 2^2 - 1 \\ 8 = 3^2 - 1 \\ 15 = 4^2 - 1\end{gathered}$$

Thus,

$$\text{Step } n = (n+1)^2 - 1$$

For Step 14:

$$\begin{gathered}(15)^2 - 1 \\ 225 - 1 = 224\end{gathered}$$

Answer: $\boxed{224}$

G. Long Answer Type Questions

1(i). Find 32×10001

$$\begin{gathered}32(10000 + 1) \\ 320000 + 32 \\ = 320032\end{gathered}$$

Answer: $\boxed{320032}$

1(ii). Find 102×1001

$$\begin{gathered}102(1000 + 1) \\ 102000 + 102 \\ = 102102\end{gathered}$$

Answer: $\boxed{102102}$

1(iii). Find 145×11

$$145 \times 11$$
$$= 1450 + 145$$
$$= 1595$$

Answer: $\boxed{1595}$

1(iv). Find 3246×101

$$3246(100 + 1)$$
$$324600 + 3246$$
$$= 327846$$

Answer: $\boxed{327846}$

2(i). Find 47^2

$$(50 - 3)^2$$
$$= 2500 - 300 + 9$$
$$= 2209$$

Answer: $\boxed{2209}$

2(ii). Find 497×503

$$= (500 - 3)(500 + 3)$$

Using

$$(a - b)(a + b) = a^2 - b^2$$
$$= 500^2 - 3^2$$
$$= 250000 - 9$$
$$= 249991$$

Answer: $\boxed{249991}$

2(iii). Find $91^2 - 89^2$

Using

$$a^2 - b^2 = (a+b)(a-b)$$
$$= (91+89)(91-89)$$
$$= 180 \times 2$$
$$= 360$$

Answer: $\boxed{360}$

2(iv). Find $(x+4)^2 - (x-4)^2$

Using

$$a^2 - b^2 = (a+b)(a-b)$$
$$= [(x+4)+(x-4)][(x+4)-(x-4)]$$
$$= (2x)(8)$$
$$= 16x$$

Answer: $\boxed{16x}$

Q.3 If $m+n=48$and $m-n=24$, find the value of m^2+n^2+mn.

Step 1: Find $m$and n

Given:

$$m+n=48$$
$$m-n=24$$

Adding the equations:

$$2m=72$$
$$m=36$$

Substitute in $m+n=48$:

$$36+n=48$$
$$n=12$$

Step 2: Find $m^2 + n^2 + mn$

$$= 36^2 + 12^2 + (36 \times 12)$$
$$= 1296 + 144 + 432$$
$$= 1872$$

Answer

$$\boxed{1872}$$

Q.4 Find the shaded area if $PT = TD$

The outer figure is a square of side:

$$a + b$$

So,

$$\text{Area of square} = (a + b)^2$$

Step 1: Find the top unshaded rectangle

Given:

$$PT = TD$$

Also,

$$AD = a + b$$

Hence T is the midpoint of DP, so

$$DT = PT = \frac{a + b}{2}$$

The top unshaded rectangle extends from the left side to line VR.

Since

$$QR = b, UQ = b$$

therefore

$$UR = 2b$$

and the full height is $a + b$.

Thus the width of the right strip is

$$VC = RS = (a + b) - 2b = a - b$$

Hence the width of the top rectangle is

$$DV = 2b$$

Area of top rectangle:

$$= 2b \times \frac{a + b}{2}$$
$$= b(a + b)$$

Step 2: Find the bottom-right unshaded rectangle

Its dimensions are:

$$RS = a - b$$

and

$$RB = b$$

Area:

$$= (a - b)b$$

Step 3: Find shaded area

$$\text{Shaded Area} = (a+b)^2 - b(a+b) - b(a-b)$$
$$= (a+b)^2 - 2ab$$

Using expansion:

$$= a^2 + 2ab + b^2 - 2ab$$
$$= a^2 + b^2$$

Answer

$$\boxed{a^2 + b^2}$$

Therefore, the shaded area is

$$\boxed{a^2 + b^2 \text{ square units}}$$

Competency-Based Questions

A. Assertion–Reason Questions

1.

Assertion (A): The increase in the product of 58 and 62 when 58 is increased by 8 and 62 is increased by 4 is 760.

Reason (R): If the product of two numbers $a$and $b$is ab, then increasing $a$by $m$and $b$by $n$increases the product by $mb + an + mn$.

Verification

$$\text{Increase} = 8(62) + 4(58) + 8(4)$$
$$= 496 + 232 + 32$$
$$= 760$$

Assertion is **True**.

Reason formula is also **True**.

Reason correctly explains Assertion.

Answer: (a)

2.

Assertion (A):

$$(5-y)^2 = 25 + 10y - y^2$$

Using identity:

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(5-y)^2 = 25 - 10y + y^2$$

Assertion is **False**.

Reason is **True**.

Answer: (c)

3.

Assertion (A):

If

$$x + y = 17, x - y = 3$$

then

$$x^2 + y^2 = 149$$

Using:

$$(x+y)^2 + (x-y)^2 = 2(x^2 + y^2)$$
$$17^2 + 3^2 = 2(x^2 + y^2)$$
$$289 + 9 = 298$$
$$x^2 + y^2 = 149$$

Assertion is **True**.

Reason:

$$(a+b)^2-(a-b)^2=4ab$$

also True but does not explain Assertion.

Answer: (b)

4.

Assertion (A):

$$189^2-181^2=2960$$

Using:

$$\begin{aligned}&a^2-b^2=(a+b)(a-b)\\&=(189+181)(189-181)\\&=370\times 8\\&=2960\end{aligned}$$

Assertion True.

Reason True and explains Assertion.

Answer: (a)

B. Case Study Questions

1. Hardik's Return Gift Bags

Given:

- 3 chocolates per bag
- 3 pencils per bag
- Chocolate = ₹10 each
- Pencil = ₹5 each

(i) Cost of one gift bag

Chocolate cost:

$$3\times 10=30$$

Pencil cost:

$$3 \times 5 = 15$$

Total:

$$30 + 15 = 45$$

Answer: ₹45

(ii) Cost of 6 bags

$$45 \times 6$$
$$= 270$$

Answer: ₹270

(iii) Cost of 8 bags

$$45 \times 8$$
$$= 360$$

Answer: ₹360

---

2. Square Garden

Side of garden:

$$(2a + b)\text{ m}$$

Path width:

$$a\text{ m}$$

(i) Area of garden

$$= (2a + b)^2$$

$$= 4a^2 + 4ab + b^2$$

Answer:

$$\boxed{4a^2 + 4ab + b^2}$$

(ii) Area of path

Inner square side:

$$(2a + b) - 2a = b$$

Inner area:

$$b^2$$

Path area:

$$(2a + b)^2 - b^2$$

Using identity:

$$= 4a^2 + 4ab + b^2 - b^2$$
$$= 4a^2 + 4ab$$

Answer:

$$\boxed{4a^2 + 4ab}$$

C. Maths Booster

1. Multiplication of 47 and 23

$$47 = (40 + 7)$$
$$23 = (20 + 3)$$

× **20** **3**

40 800 120

7 140 21

Total:

$$800 + 120 + 140 + 21$$
$$= 1081$$

Answer: 1081

---

2. Number Pattern

Sequence:

$$2,\ 5,\ 10,\ 17, \ldots$$

Differences:

$$+3,\ \ +5,\ \ +7$$

Next difference:

$$+9$$

Step 5:

$$17 + 9 = 26$$

Pattern:

$$n^2 + 1$$

Check:

$$1^2 + 1 = 2$$
$$2^2 + 1 = 5$$
$$3^2 + 1 = 10$$
$$4^2 + 1 = 17$$

$$5^2 + 1 = 26$$

Answer: 26

---

3. Pattern Hunt in Squares

Given:

$$2(3^2 + 2^2) = 5^2 + 1^2$$
$$2(6^2 + 2^2) = 8^2 + 4^2$$

Next two patterns

$$2(9^2 + 2^2) = 11^2 + 7^2$$

Check:

$$2(81 + 4) = 170$$
$$121 + 49 = 170$$

✓

$$2(12^2 + 2^2) = 14^2 + 10^2$$

Check:

$$2(144 + 4) = 296$$
$$196 + 100 = 296$$

✓

General Formula

$$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$$

Answer:

$$\boxed{2(a^2 + b^2) = (a + b)^2 + (a - b)^2}$$

---

4. Without Multiplying, Decide Which is Larger

(i) $\mathbf{18 \times 22}$or $\mathbf{20 \times 20}$

$$18 \times 22 = (20 - 2)(20 + 2)$$
$$= 20^2 - 2^2$$
$$= 400 - 4$$
$$= 396$$
$$20 \times 20 = 400$$

Therefore

$$\boxed{20 \times 20}$$

is larger.

(ii) $\mathbf{47 \times 53}$or $\mathbf{50 \times 50}$

$$47 \times 53 = (50 - 3)(50 + 3)$$
$$= 50^2 - 3^2$$
$$= 2500 - 9$$
$$= 2491$$
$$50 \times 50 = 2500$$

Therefore

$$\boxed{50 \times 50}$$

is larger.

Chapter 7 Proportional Reasoning 1

NCERT CORNER

IN TEXT

IN TEXT –

1

(i) Teacher : Student ratio in my school

Given:

Teachers = 5

Students = 170

$$\text{Teacher : Student} = 5:170$$

Divide both terms by 5:

$$= 1:34$$

Answer: $\boxed{1:34}$

(ii) Teacher : Student ratio in your school

Let:

- Number of teachers = T
- Number of students = S

Then

$$\boxed{T:S}$$

(Substitute the actual numbers from your school.)

(iii) Is the ratio proportional to the one in my school?

Compare your ratio with $1:34$.

- If equal → **Yes**
- If not equal → **No**

2

Measure the blackboard.

Example:

Width = 240 cm

Height = 120 cm

$$240:120$$

Divide by 120:

$$2:1$$

Answer: $\boxed{2:1}$

(Use your classroom measurements.)

3

Draw a rectangle proportional to the blackboard ratio.

If the blackboard ratio is $2:1$, possible rectangles are:

- 10 cm × 5 cm
- 12 cm × 6 cm
- 14 cm × 7 cm

All have ratio

$$2:1$$

Answer: Any rectangle having the same width : height ratio as the blackboard.

4

Compare the rectangles drawn by classmates.

Answer:

Yes, all rectangles having the same width : height ratio are proportional and therefore look alike, though their sizes may be different.

IN TEXT

1.
Filter coffee is a beverage made by mixing coffee decoction with milk. Manjunath usually mixes 15 mL of coffee decoction with 35 mL of milk to make one cup of filter coffee in his coffee shop. In this case, we can say that the ratio of coffee decoction to milk is 15 : 35.

If customers want 'stronger' filter coffee. Manjunath mixes 20 mL of the decoction with 30 mL of milk. The ratio here is 20 : 30. Why is this coffee stronger?

(i) And when they want 'lighter' filter coffee, he mixes 10 mL of coffee and 40 mL of milk, making the ratio 10 : 40. Why is this coffee lighter?

2. The following table shows the different ratios in which Manjunath mixes coffee decoction with milk. Write in the last column if the coffee is stronger or lighter than the regular coffee. (Pages 164-165)

Coffee Decoction (in mL)	Milk (in mL)	Regular/Strong/ Light
300	600	
150	500	
200	400	
24	56	
100	300	

Solution:

Solution:

In one cup of regular filter coffee: Coffee decoction = 15 mL

Milk = 35 mL

∴ Ratio of coffee decoction to milk = 15 : 35 = 3 : 7

Here, 3 + 7 = 10

∴ Coffee decoction in 10 mL filter coffee = 3 mL

∴ Coffee decoction in 100 mL filter coffee = $\frac{3}{10}$ × 100 = 30 mL

In one cup of stronger filter coffee:

Coffee decoction = 20 mL

Milk = 30 mL

∴ Ratio of coffee decoction to milk = 20 : 30 = 2 : 3

Here, 2 + 3 = 5.

∴ Coffee decoction in 5 mL filter coffee = 2 mL

∴ Coffee decoction in 100 mL filter coffee = $\frac{2}{5}$ × 100 = 40 mL

Since 40 mL > 30 mL, the latter coffee is stronger.

(i) In a cup of lighter filter coffee:

Coffee decoction = 10 mL

Milk = 40 mL

∴ Ratio of coffee decoction to milk = 10 : 40 = 1 : 4

Here, 1 + 4 = 5.

∴ Coffee decoction in 5 mL filter coffee = 1 mL

∴ Coffee decoction in 100 mL filter coffee = $\frac{1}{5}$ × 100 = 20 mL

Since 20 mL < 30 mL, the third type of filter coffee is lighter.

(ii)

S. No.	Coffee Decoction (in mL)	Milk (in mL)	Regular/ Stronger/ Lighter
1	300	600	Stronger
2	150	500	Lighter
3	200	400	Stronger
4	24	56	Regular
5	100	300	Lighter

S.No.1. Here 300 + 600 = 900

∴ Coffee decoction in 900 mL filter coffee = 300

∴ Coffee decoction in 100 mL filter coffee = $\frac{300}{900}$ × 100

= $\frac{100}{3}$

= $33\frac{1}{3}$ mL

Since $33\frac{1}{3}$ > 30, this filter coffee is stronger.

S.No.2. Here 150 + 500 = 650

∴ Coffee decoction in 650 mL filter coffee = 150

∴ Coffee decoction in 100 mL filter coffee = $\frac{150}{650}$ × 100

= $\frac{300}{13}$

= $23\frac{1}{13}$ mL

Since $23\frac{1}{13}$ < 30, this filter coffee is lighter.

S.No. 3. Here 200 + 400 = 600

∴ Coffee decoction in 600 mL filter coffee = 200

∴ Coffee decoction in 100 mL filter coffee = $\frac{200}{600}$ × 100

= $\frac{100}{3}$

= $33\frac{1}{3}$ mL

Since $33\frac{1}{3}$ > 30, this filter coffee is stronger.

S.No. 4. Here 24 + 56 = 80

∴ Coffee decoction in 80 mL filter coffee = 24 mL

∴ Coffee decoction in 100 mL filter coffee = $\frac{24}{80}$ × 100 = 30

Since 30 = 30, this filter coffee is regular.

S.No. 5. Here 100 + 300 = 400

∴ Coffee decoction in 400 mL filter coffee = 100 mL

∴ Coffee decoction in 100 mL filter coffee = $\frac{100}{400}$ × 100 = 25 mL

Since 25 < 30, this filter coffee is lighter.

Figure It Out

Figure It Out – Q.1

Check whether the two ratios are equal.

(i) 4 : 7 :: 12 : 21

$$\frac{4}{7} = \frac{12}{21} = \frac{4}{7}$$

✅ **True**

(ii) 8 : 3 :: 24 : 6

$$\frac{8}{3} \neq \frac{24}{6}$$
$$\frac{8}{3} \neq 4$$

❌ **False**

(iii) 7 : 12 :: 12 : 7

$$\frac{7}{12} \neq \frac{12}{7}$$

❌ **False**

(iv) 21 : 6 :: 35 : 10

$$\frac{21}{6} = \frac{35}{10} = \frac{7}{2}$$

✅ **True**

(v) 12 : 18 :: 28 : 12

$$\frac{12}{18} = \frac{2}{3}$$
$$\frac{28}{12} = \frac{7}{3}$$

❌ **False**

(vi) 24 : 8 :: 9 : 3

$$\frac{24}{8} = 3$$

$$\frac{9}{3} = 3$$

✅ **True**

Answer:

True proportions are:

$$\boxed{(i),\ (iv),\ (vi)}$$

Q.2 Give 3 ratios proportional to 4 : 9

Multiply both terms by the same number.

× 2:

$$8 : 18$$

× 3:

$$12 : 27$$

× 4:

$$16 : 36$$

Answer:

$$\boxed{8 : 18,\ 12 : 27,\ 16 : 36}$$

Q.3 Fill in the missing numbers for ratios proportional to 18 : 24

$$18 : 24 = 3 : 4$$

So each ratio must be equivalent to $3 : 4$.

(i)

$$3:\boxed{4}$$

(ii)

$$12:\boxed{16}$$

(iii)

$$20:\boxed{\frac{80}{3}}$$

(Not a whole number. The question likely expects equivalent ratios.)

Using $3:4$:

$$20:x$$
$$\frac{20}{x}=\frac{3}{4}$$
$$x=\frac{80}{3}$$

(iv)

$$27:\boxed{36}$$

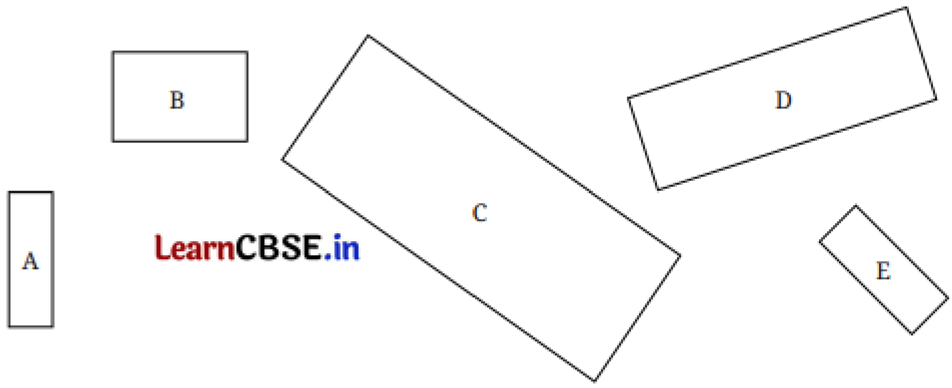
Answer:

$$\boxed{3:4,12:16,20:\frac{80}{3},27:36}$$

(Many editions actually print 24 instead of 20 so that all answers are whole numbers.)

Question 4.
Look at the following rectangles. Which rectangles are similar to each other? You can verify this by measuring the width and height using a scale and

comparing their ratios.



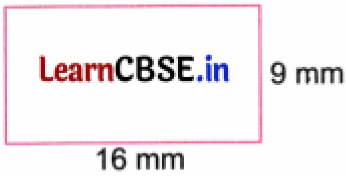
Solution:
Using a scale, we measure the width and height of given rectangles.
Here, the ratio 'Width : Height' for given rectangles A, B, C, D, and E are respectively 1 : 3, 3 : 2, 9 : 4, 7 : 2 and 3 : 1.
These ratios are all distinct. The ratios of A and E are 1 : 3 and 3 : 1 respectively.

Rectangle	Width (in mm)	Height (in mm)	Ratio = $\frac{\text{Width}}{\text{Height}}$
A	5	15	$\frac{5}{15} = \frac{1}{3}$
B	15	10	$\frac{15}{10} = \frac{3}{2}$
C	45	20	$\frac{45}{20} = \frac{9}{4}$
D	35	10	$\frac{35}{10} = \frac{7}{2}$
E	15	5	$\frac{15}{5} = \frac{3}{1}$

∴ For A and E, one side is 3 times the other side.
∴ Only rectangles A and E are similar.

Question 5.
Look at the following rectangle. Can you draw a smaller rectangle and a bigger rectangle with the same width-to-height ratio in your notebook? Compare your rectangles with your classmates' drawings. Are all of them the same? If they are different from yours, can you think why? Are they wrong?

Solution:
For the given rectangle;
Width = 32 mm and height = 18 mm
∴ Ratio is 32 : 18.
We shall draw smaller and bigger rectangles and similar to the given rectangle by considering different 'factors of change'.
Let the factor of change be $\frac{1}{2}$
$\frac{1}{2} \times 32 = 16$ mm
and New height = $\frac{1}{2} \times 18 = 9$ mm
A new, similar rectangle is shown in the figure.

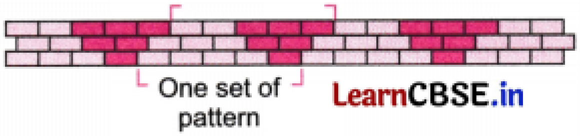


Let 'factor of change' be 2.
∴ New width = 2 × 32 = 64 mm and new height = 2 × 18 = 36 mm

A new, similar rectangle is shown in the figure. The rectangles drawn by other classmates are all different, but they are all similar to the given rectangle.

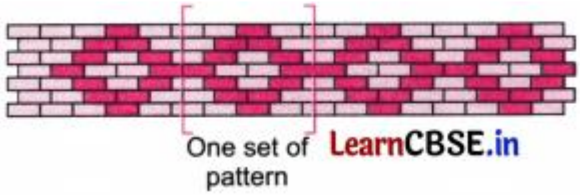
Question 6.
The following figure shows a small portion of a long brick wall with patterns made using coloured bricks. Each wall continues this pattern throughout the wall. What is the ratio of grey bricks to coloured bricks? Try to give the ratios in their simplest form.

Solution:
(a) We consider one set of patterns in the given wall.



Number of grey bricks in one set of pattern = 2 + 3 + 4 = 9
Number of coloured bricks in one set of pattern = 3 + 2 + 1 = 6
∴ Ratio of grey bricks to coloured bricks = 9 : 6
We have 9 : 6 = 3 : 2
∴ Ratio in the simplest form = 3 : 2

(b) We use one set of patterns on the given wall



Number of grey bricks in one set of pattern

$$= \left(\frac{1}{2} + 1 + 1 + \frac{1}{2}\right) + (1 + 1) + \left(\frac{1}{2} + 1 + \frac{1}{2}\right) + (1 + 1) + \left(\frac{1}{2} + 1 + \frac{1}{2}\right) + (1 + 1) + \left(\frac{1}{2} + 1 + 1 + \frac{1}{2}\right)$$

= 3 + 2 + 2 + 2 + 2 + 2 + 3
= 16
Number of coloured bricks in one set of pattern = 1 + (1 + 1) + (1 + 1) + (1 + 1) + (1 + 1) + (1 + 1) + 1
= 1 + 2 + 2 + 2 + 2 + 2 + 1
= 12
∴ Ratio of grey bricks to coloured bricks = 16 : 12
We have 16 : 12 = 4 : 3
∴ Ratio in the simplest form = 4 : 3.

Q.7 Let us draw some human figures...

This is an **activity-based question**. The answer depends on the measurements of the figure drawn.

For a sample figure:

- Head = 2 cm
- Torso = 4 cm
- Arms = 4 cm

- Legs = 6 cm

Then,

$$\text{Head : Torso} = 2:4 = 1:2$$
$$\text{Torso : Arms} = 4:4 = 1:1$$
$$\text{Torso : Legs} = 4:6 = 2:3$$

Answer (Sample)

Head : Torso = **1 : 2**

Torso : Arms = **1 : 1**

Torso : Legs = **2 : 3**

(Your answer may vary according to your drawing.)

Q.8 Draw a figure with head, torso, arms and legs with equivalent ratios as above.

Answer

Yes, the drawing will look more realistic if the body parts are in proportional ratios.

For example:

If

$$\text{Head : Torso} = 1:2$$

then doubling both lengths gives

$$2:4$$

or tripling gives

$$3:6$$

The shape remains similar because the proportions are unchanged.

Therefore

$$\boxed{\text{Yes, the figure looks more realistic when body parts are proportional.}}$$

Why?

Because in real human bodies, the sizes of the head, torso, arms and legs follow approximately fixed proportions. When these proportions are maintained, the figure appears balanced and natural. When they are not maintained, the figure may look distorted or unrealistic.

Figure It Out

1. Divide ₹4500 into two parts in the ratio 2 : 3

Total parts

$$2 + 3 = 5$$

Value of 1 part

$$4500 \div 5 = 900$$

First part

$$2 \times 900 = 1800$$

Second part

$$3 \times 900 = 2700$$

Answer: ₹1800 and ₹2700

---

2. Acid : Water = 1 : 5, Total solution = 240 ml

Total parts

$$1 + 5 = 6$$

One part

$$240 \div 6 = 40$$

Acid

$$1 \times 40 = 40 \text{ ml}$$

Water

$$5 \times 40 = 200 \text{ ml}$$

Answer: Acid = 40 ml, Water = 200 ml

---

3. Blue : Yellow = 3 : 5

For 40 ml paint:

Total parts

$$3 + 5 = 8$$

One part

$$40 \div 8 = 5$$

Blue

$$3 \times 5 = 15 \text{ ml}$$

Yellow

$$5 \times 5 = 25 \text{ ml}$$

After adding 20 ml yellow:

Yellow

$$25 + 20 = 45 \text{ ml}$$

New ratio

$$15: 45 = 1: 3$$

Answer: Blue = 15 ml, Yellow = 25 ml; New ratio = **1 : 3**

---

4. Rice : Urad dal = 2 : 1

Total cups = 6

Total parts

$$2 + 1 = 3$$

One part

$$6 \div 3 = 2$$

Rice

$$2 \times 2 = 4 \text{ cups}$$

Urad dal

$$1 \times 2 = 2 \text{ cups}$$

Answer: Rice = 4 cups, Urad dal = 2 cups

---

5. Red : Yellow = 3 : 5

Assume one bucket contains 8 units.

Red = 3 units

Yellow = 5 units

Another bucket of yellow paint added = 8 units

New yellow

$$5 + 8 = 13$$

New ratio

$$3 : 13$$

Answer: 3 : 13

Figure It Out

1. Orange juice : Apple juice

$$600:900$$

Divide by 300

$$2:3$$

Answer: 2 : 3

---

2. Buses required

Last year:

3 buses for 162 people

People per bus

$$162 \div 3 = 54$$

This year:

$$204 \div 54 = 3.78$$

Need 4 buses.

Will buses be full?

$$4 \times 54 = 216$$

Seats left

$$216 - 204 = 12$$

Answer: 4 buses are needed. No, they will not be full.

---

3. Which city is more crowded?

Delhi:

$$\frac{30,000,000}{1484} \approx 20216$$

people per sq km

Mumbai:

$$\frac{20,000,000}{550} \approx 36364$$

people per sq km

Since

$$36364 > 20216$$

Answer: Mumbai is more crowded.

---

4. Crane problem

Height = 155 cm

Neck : Rest of body = 4 : 6

Total parts

$$4 + 6 = 10$$

One part

$$155 \div 10 = 15.5$$

Neck height

$$4 \times 15.5 = 62$$

Answer: 62 cm

---

5. Saffron problem

$$2\frac{1}{2} = \frac{5}{2}$$
$$\frac{5}{2} \text{ palas cost } \frac{3}{7} \text{ niskas}$$

Quantity for 1 niska

$$\frac{5}{2} \div \frac{3}{7} = \frac{5}{2} \times \frac{7}{3} = \frac{35}{6}$$

Quantity for 9 niskas

$$\frac{35}{6} \times 9 = \frac{105}{2} = 52.5$$

Answer: 52½ palas of saffron

---

6. Harmain's age

Present ages:

Harmain = 1 year

Brother = 5 years

After $x$years:

$$\frac{1+x}{5+x} = \frac{1}{2}$$
$$2(1+x) = 5+x$$
$$2+2x = 5+x$$
$$x = 3$$

Harmain's age then

$$1+3=4$$

Answer: 4 years

---

7. Gold and water

Gold : Water = 37 : 2

If 1 litre water = 1 kg

$$2 \text{ parts} = 1 \text{ kg}$$
$$1 \text{ part} = 0.5 \text{ kg}$$

Gold mass

$$37 \times 0.5 = 18.5$$

Answer: 18.5 kg

---

8. Manure required

Plot area

$$200 \times 500 = 100000 \text{ sq ft}$$

1 acre

$$43560 \text{ sq ft}$$

Area in acres

$$\frac{100000}{43560} \approx 2.295$$

Manure needed

$$2.295 \times 10 \approx 22.95$$

Answer: Approximately **23 tonnes**

---

9. Bucket filling time

500 ml takes 15 s

10 litres

$$= 10000 \text{ ml}$$

$$10000 \div 500 = 20$$

Time

$$20 \times 15 = 300 \text{ s}$$
$$300 \text{ s} = 5 \text{ minutes}$$

Answer: 5 minutes

---

10. Cost of 2400 sq ft land

1 acre

$$= 43560 \text{ sq ft}$$

Cost of 1 sq ft

$$\frac{1500000}{43560} \approx 34.435$$

Cost of 2400 sq ft

$$34.435 \times 2400 \approx 82644.6$$

Answer: ₹82,645 (approx.)

11. Tractor and Oxen Problem

Given:

- A tractor can plough the same area **4 times faster** than a pair of oxen.
- A pair of oxen takes **6 hours** to plough **1 acre**.
- Field area = **20 acres**

Step 1: Time taken by oxen for 20 acres

If 1 acre takes 6 hours,

20 acres will take

$$20 \times 6 = 120 \text{ hours}$$

Answer:

Oxen will take 120 hours.

Step 2: Time taken by tractor

Tractor is 4 times faster.

Therefore, time required becomes

$$\frac{120}{4} = 30 \text{ hours}$$

Answer:

Tractor will take 30 hours.

Final Answer

- Time taken by oxen = **120 hours**
- Time taken by tractor = **30 hours**

---

12. Cost of Metals in a ₹10 Coin

Given:

- Copper : Nickel = **3 : 1**
- Mass of coin = **7.74 g**
- Cost of copper = ₹906 per kg
- Cost of nickel = ₹1341 per kg

Step 1: Find mass of copper and nickel

Total ratio parts

$$3 + 1 = 4$$

Copper mass

$$\frac{3}{4} \times 7.74$$
$$= 5.805 \text{ g}$$

Nickel mass

$$\frac{1}{4} \times 7.74$$
$$= 1.935 \text{ g}$$

Step 2: Convert grams to kilograms

Copper

$$5.805g = \frac{5.805}{1000} = 0.005805kg$$

Nickel

$$1.935g = \frac{1.935}{1000} = 0.001935kg$$

Step 3: Cost of copper

$$0.005805 \times 906$$
$$= ₹5.25933$$

Step 4: Cost of nickel

$$0.001935 \times 1341$$
$$= ₹2.595835$$

Step 5: Total cost

$$5.25933 + 2.595835$$
$$= ₹7.855165$$
$$\approx ₹7.86$$

Final Answer

The cost of metals in one ₹10 coin ≈ ₹7.86

(Direct Method)

Average cost per kg of alloy:

$$\frac{3(906) + 1(1341)}{4}$$
$$= \frac{2718 + 1341}{4}$$
$$= \frac{4059}{4}$$
$$= 1014.75 \text{ ₹/kg}$$

Mass of coin

$$7.74g = 0.00774kg$$

Cost

$$1014.75 \times 0.00774$$
$$= ₹7.85 \approx ₹7.86$$

Practice Time 7.1

1. Ratio between 240 minutes and 360 minutes

$$240:360$$
$$= 2:3$$

✅ Answer: 2 : 3

---

2. If $a : b = 2 : 3$, find $(3a + 2b) : (2a + 3b)$

Let

$$a = 2k, b = 3k$$
$$3a + 2b = 6k + 6k = 12k$$
$$2a + 3b = 4k + 9k = 13k$$

Ratio:

$$12 : 13$$

✅ Answer: 12 : 13

---

3. Numbers in ratio 3 : 4 and sum 9

Let numbers be

$$3x,\ 4x$$
$$3x + 4x = 9$$
$$7x = 9$$
$$x = \frac{9}{7}$$

Numbers:

$$\frac{27}{7}, \frac{36}{7}$$

✅ Answer: $\frac{27}{7}$and $\frac{36}{7}$

4.

Milk : Water = 30 : 20

Add 10 L milk

Milk = 40 L

Water = 20 L

$$40:20 = 2:1$$

✅ Answer: 2 : 1

5.

Boys : Girls = 5 : 2

Let numbers be

$$5x,\ 2x$$

After adding girls:

$$\begin{aligned} &5x:(2x+y) = 10:7 \\ &35x = 20x + 10y \\ &15x = 10y \\ &y = \frac{3x}{2} \end{aligned}$$

Since total girls initially = 2x,

girls added = $\frac{3}{4}$of existing girls.

✅ Answer: Add ¾ of the original number of girls.

6.

$$7.8\text{ cm}: 8\text{ km}$$

Convert km to cm:

$$\begin{aligned} &8\text{ km} = 800000\text{ cm} \\ &7.8: 800000 \end{aligned}$$

Multiply by 10:

$$\begin{aligned} 78&: 8000000 \\ 39&: 4000000 \end{aligned}$$

✅ Answer: 39 : 4000000

7.

Length = 80 m

Breadth = 4000 cm = 40 m

Area:

$$
\begin{aligned}
&80 \times 40 \\
&= 3200\, m^2
\end{aligned}
$$

Convert to square feet:

$$
\begin{aligned}
&3200 \times 10.764 \\
&= 34444.8\, ft^2
\end{aligned}
$$

✅ Answer: 34,444.8 sq ft

8.

$$C = \frac{5}{9}(F - 32)$$

For $F = 50^\circ$

$$
\begin{aligned}
C &= \frac{5}{9}(50 - 32) \\
&= \frac{5}{9} \times 18 \\
&= 10
\end{aligned}
$$

✅ Answer: $10^\circ C$

Practice Time 7.2

1. Which are in proportion?

(i)

$$15:60 = 1:4$$
$$12:48 = 1:4$$

✅ In proportion

(ii)

$$75:25 = 3:1$$
$$90:30 = 3:1$$

✅ In proportion

(iii)

$$25:5 = 5:1$$
$$30:5 = 6:1$$

❌ Not in proportion

Answer: (i) and (ii)

---

2. Three ratios proportional to 5 : 8

Multiply by 2, 3, 4

$$10:16, 15:24, 20:32$$

✅ Answer

3.

$$n:15::4:30$$
$$\frac{n}{15} = \frac{4}{30}$$
$$30n = 60$$
$$n = 2$$

✅ Answer: 2

4.

$$8 : a :: 4 : 20$$
$$\frac{8}{a} = \frac{4}{20}$$
$$160 = 4a$$
$$a = 40$$

✅ Answer: 40

5.

Numbers 5, 11, 19, 37

Let x be added to each.

$$\frac{5+x}{11+x} = \frac{19+x}{37+x}$$

Cross multiply:

$$(5+x)(37+x) = (11+x)(19+x)$$
$$185 + 42x + x^2 = 209 + 30x + x^2$$
$$12x = 24$$
$$x = 2$$

✅ Answer: 2

6.

90 km in 150 min

Speed:

$$90/150 = 0.6\ km/min$$

Distance in 250 min:

$$0.6 \times 250$$
$$= 150$$

✅ Answer: 150 km

EXAM TIME

MCQ 1

The ratio of 65 minutes to 104 minutes is

$$65:104$$

Divide both terms by 13:

$$\frac{65}{13}:\frac{104}{13}$$
$$5:8$$

✅ **Answer: (c) 5 : 8**

MCQ 2

Given

$$x:y = 5:6$$

Let

$$x = 5k, y = 6k$$

Substitute:

$$\frac{6x + 4y}{6x - 4y}$$
$$= \frac{6(5k) + 4(6k)}{6(5k) - 4(6k)}$$

$$
\begin{aligned}
&= \frac{30k + 24k}{30k - 24k} \\
&= \frac{54k}{6k} \\
&= 9
\end{aligned}
$$

✅ **Answer: (b) 9**

MCQ 3

Let side = a

Perimeter of equilateral triangle

$$= 3a$$

Perimeter of square

$$= 4a$$

Ratio

$$
\begin{aligned}
&3a : 4a \\
&3 : 4
\end{aligned}
$$

✅ **Answer: (a) 3 : 4**

MCQ 4

Given

$$26X = 65Y$$

Divide by 13:

$$
\begin{aligned}
&2X = 5Y \\
&\frac{X}{Y} = \frac{5}{2} \\
&X : Y = 5 : 2
\end{aligned}
$$

✅ **Answer: (d) 5 : 2**

MCQ 5

Given

$$(3a+2b):(5a+3b) = 18:29$$

Let

$$a:b = 3:4$$

Check:

$$3a+2b = 3(3)+2(4) = 17$$
$$5a+3b = 5(3)+3(4) = 27$$

Not 18:29

Try

$$a:b = 4:3$$
$$3a+2b = 3(4)+2(3) = 18$$
$$5a+3b = 5(4)+3(3) = 29$$

Ratio =

$$18:29$$

✅ **Answer: (b) 4 : 3**

MCQ 6

Pass : Fail = 3 : 1

Let

$$P = 3x, F = x$$

If 20 less had passed and 10 less appeared:

New pass

$$3x - 20$$

New fail

$$x + 10$$

New ratio

$$5:1$$
$$\frac{3x - 20}{x + 10} = 5$$
$$3x - 20 = 5x + 50$$
$$2x = -70$$
$$x = 35$$

Total students

$$3x + x = 4x$$
$$= 140$$

But 10 fewer appeared:

$$140 - 60 = 80$$

✅ **Answer: (a) 80**

MCQ 7

Ratio = 3 : 7

Smaller piece = 3 parts = 60 m

One part

$$= 60 \div 3 = 20$$

Total parts = 10

Total length

$$= 10 \times 20$$
$$= 200m$$

✅ **Answer: (c) 200 m**

MCQ 8

Ratio = 5 : 3

Total parts = 8

Sum = 128

One part

$$128 \div 8 = 16$$

Smaller number

$$3 \times 16 = 48$$

✅ **Answer: (b) 48**

MCQ 9

Total mixture = 30 kg

Ratio 2 : 1

Sand

$$= \frac{2}{3} \times 30 = 20kg$$

Cement

$$= \frac{1}{3} \times 30 = 10kg$$

Add x kg sand.

$$\frac{20+x}{10} = 3$$
$$20 + x = 30$$
$$x = 10$$

✅ **Answer: (c) 10 kg**

MCQ 10

8 L = 8000 mL

Ratio

$$8000{:}\,1600$$
$$5{:}\,1$$

✅ **Answer: (c) 5 : 1**

MCQ 11

Breadth = 15 cm

Length = 3 × 15

$$= 45cm$$

Perimeter

$$= 2(l+b)$$
$$= 2(45+15)$$
$$= 120cm$$

Convert to feet

$$120 \div 30.48$$
$$= 3.9372 \text{ ft}$$

✅ **Answer: (b) 3.9372 feet**

MCQ 12

Formula:

$$F = \frac{9}{5}C + 32$$
$$= \frac{9}{5}(30.5) + 32$$
$$= 54.9 + 32$$
$$= 86.9^\circ F$$

✅ **Answer: (b) 86.9°**

MCQ 13

Check proportionality:

(a)

$$40:8 = 5$$
$$30:6 = 5$$

Proportion ✓

(b)

$$64:16 = 4$$
$$32:8 = 4$$

Proportion ✓

(c)

$$102:17 = 6$$
$$76:19 = 4$$

Not equal ✗

(d)

$$105:15 = 7$$
$$63:9 = 7$$

Proportion ✓

✅ **Answer: (c)**

MCQ 14

Equivalent ratio of

$$17:21$$

Multiply by 9:

$$153:189$$

✅ **Answer: (a)**

MCQ 15

$$7:28::5:a$$
$$\frac{7}{28} = \frac{5}{a}$$
$$\frac{1}{4} = \frac{5}{a}$$
$$a = 20$$

✅ **Answer: (b) 20**

MCQ 16

If

$$a:b = c:d$$

then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

✅ **Answer: (d)**

MCQ 17

Add x:

$$\frac{5+x}{13+x} = \frac{6+x}{15+x}$$

Cross multiply:

$$(5+x)(15+x) = (6+x)(13+x)$$
$$75 + 20x + x^2 = 78 + 19x + x^2$$
$$x = 3$$

✅ **Answer: (b) 3**

MCQ 18

(a)

$$81:9 = 9$$
$$64:8 = 8$$

Not proportional

(b)

$$21:84 = 1:4$$
$$25:100 = 1:4$$

Proportional ✓

(c)

$$19:95 = 1:5$$
$$17:85 = 1:5$$

Proportional ✓

✅ **Answer: (d) Both (b) and (c)**

MCQ 19

Given ratio

$$9:13$$

Check options:

(a)

$$108:156 = 9:13$$

✓

(b)

$$117:169 = 9:13$$

✓

(c)

$$126:195 = 42:65$$

✗

(d)

$$81:117 = 9:13$$

✓

✅ **Answer: (c)**

MCQ 20

9 dolls → ₹333

1 doll

$$= 333 \div 9 = 37$$

16 dolls

$$= 37 \times 16$$
$$= 592$$

✅ **Answer: (d) ₹592**

B. Fill in the Blanks

1. The simplest form of 95 : 114 is ______

Find HCF of 95 and 114.

$$95 = 5 \times 19$$
$$114 = 6 \times 19$$

HCF = 19

Divide both terms by 19:

$$95 : 114 = 5 : 6$$

✅ **Answer: 5 : 6**

---

2. The ratio of 36 hours to 54 hours is ______

$$36 : 54$$

Divide by 18:

$$2 : 3$$

✅ **Answer: 2 : 3**

---

3. If $a : b = 2 : 5$, then the value of $3a : 2b$is ______

Given

$$a : b = 2 : 5$$

Let

$$a = 2k, b = 5k$$

Then

$$\begin{aligned} &3a : 2b \\ &= 3(2k) : 2(5k) \\ &= 6k : 10k \\ &= 3 : 5 \end{aligned}$$

✅ **Answer: 3 : 5**

---

4. The ratio of 16 days to 48 hours is ______

Convert both into same units.

$$16 \text{ days} = 16 \times 24 = 384 \text{ hours}$$

Ratio:

$$\begin{aligned} &384 : 48 \\ &= 8 : 1 \end{aligned}$$

✅ **Answer: 8 : 1**

---

5. If $3 : 6 :: x : 36$, then the value of x is ______

$$\frac{3}{6} = \frac{x}{36}$$

$$\frac{1}{2} = \frac{x}{36}$$

Cross multiply:

$$x = 18$$

✅ **Answer: 18**

---

6. If the cost of 16 calculators is ₹1280, then the cost of 36 such calculators is ______

Cost of 1 calculator

$$1280 \div 16 = 80$$

Cost of 36 calculators

$$80 \times 36$$
$$= 2880$$

✅ **Answer: ₹2880**

C. True / False

1.

If both terms of a ratio are multiplied by the same quantity, ratio remains unchanged.

Example:

$$2:3 = 4:6$$

True.

✅ **True**

2.

A ratio has a unit.

Ratios compare like quantities, so units cancel.

Example:

$$20cm : 10cm = 2 : 1$$

No unit remains.

✅ **False**

3.

₹44000 : ₹66000

$$44000 : 66000$$
$$= 2 : 3$$

Statement says 4 : 3.

Wrong.

✅ **False**

4.

192 divided in ratio 29 : 35

Total parts

$$29 + 35 = 64$$

First part

$$192 \times \frac{29}{64}$$
$$= 87$$

✅ **True**

5.

56, 112, 64, 192

Check:

$$56:112 = 1:2$$
$$64:192 = 1:3$$

Not equal.

✅ **False**

6.

$$9:11$$

and

$$117:143$$
$$117 = 9 \times 13$$
$$143 = 11 \times 13$$

Ratios are equal.

✅ **True**

7.

90,18,120,x in proportion

$$90:18 = 120:x$$
$$90x = 18 \times 120$$
$$x = \frac{2160}{90}$$
$$x = 24$$

Statement says 22.

✅ **False**

Match the Columns

1.

Column I	Simplest Form	Column II
(a) 35 : 49	5 : 7	(iii)
(b) 63 : 54	7 : 6	(i)
(c) 64 : 40	8 : 5	(iv)
(d) 48 : 42	8 : 7	(ii)

✅ Answer:

(a) → (iii)

(b) → (i)

(c) → (iv)

(d) → (ii)

2.

(a)

$$3:6::5:a$$
$$\frac{3}{6}=\frac{5}{a}$$
$$a=10$$

→ (iv)

(b)

$$\frac{10}{20}=\frac{20}{x}$$
$$10x=400$$
$$x=40$$

→ (i)

(c)

$$10:60::m:90$$
$$\frac{10}{60}=\frac{m}{90}$$
$$m=15$$

→ (ii)

(d)

$$n:81::2:18$$
$$\frac{n}{81}=\frac{2}{18}$$
$$n=9$$

→ (iii)

✅ Final Matching

(a) → (iv)

(b) → (i)

(c) → (ii)

(d) → (iii)

Very Short Answer Questions

1.

If

$$a:b=3:4$$

then

$$4a:3b$$

$$
\begin{aligned}
&= 4(3):3(4)\\
&= 12:12\\
&= 1:1
\end{aligned}
$$

✅ Answer: **1 : 1**

2.

$$\frac{4}{5} \times \frac{\square}{5} = \frac{20}{\square}$$

To make numerator 20,

$$4 \times 5 = 20$$

So first box = 5

Denominator:

$$5 \times 5 = 25$$

Second box = 25

✅ Answer:

5 and 25

3(a)

600 g : 1 kg

$$
\begin{aligned}
&600:1000\\
&3:5
\end{aligned}
$$

✅ Answer: **3 : 5**

3(b)

2 cm : 4 m

$$2:400$$
$$1:200$$

✅ Answer: **1 : 200**

4(i)

70 cm : 1 m

$$70:100$$
$$7:10$$

✅ Answer: **7 : 10**

4(ii)

50 paise : ₹2

$$50:200$$
$$1:4$$

✅ Answer: **1 : 4**

5.

Equivalent ratios of 3 : 8

Multiply by 2 and 3

$$6:16,9:24$$

✅ Answer: **6 : 16 and 9 : 24**

6.

$$10:40=1:4$$
$$25:100=1:4$$

Ratios are equal.

✅ Answer: **Yes, they are in proportion.**

7.

Check

$$10:15 = \frac{2}{3}$$
$$20:30 = \frac{2}{3}$$

Ratios are equal.

✅ Answer: **Yes**

Short Answer Questions

1.

Boys = 20

Girls = 40

$$20:40$$
$$1:2$$

✅ Answer: **1 : 2**

2.

Ram = 210 m

Mohan = 180 m

$$180:210$$
$$6:7$$

✅ Answer: **6 : 7**

3.

Khoya : Sugar

$$7:2$$

Total parts = 9

Barfi = 18 kg

One part

$$18 \div 9 = 2$$

Khoya

$$7 \times 2 = 14$$

✅ Answer: **14 kg**

4.

Total length = 56 cm

Ratio = 2 : 5

Total parts = 7

One part

$$56 \div 7 = 8$$

Lengths:

$$2 \times 8 = 16$$
$$5 \times 8 = 40$$

✅ Answer: **16 cm and 40 cm**

5.

80 cm : 1.2 m

$$80:120$$
$$2:3$$

✅ Answer: **2 : 3**

6.

$$C = \frac{5}{9}(F - 32)$$
$$= \frac{5}{9}(34.7 - 32)$$
$$= \frac{5}{9}(2.7)$$
$$= 1.5$$

✅ Answer: **1.5°C**

---

7. Which pair of ratios are equal?

(i)

$$\frac{2}{3}, \frac{4}{6}$$
$$\frac{4}{6} = \frac{2}{3}$$

✅ Equal

(ii)

$$\frac{8}{4} = 2$$
$$\frac{2}{1} = 2$$

✅ Equal

(iii)

$$\frac{4}{5}$$

and

$$\frac{12}{20} = \frac{3}{5}$$

❌ Not equal

8.

8 hectares ⟶ 360 quintals

1 hectare ⟶

$$360 \div 8 = 45$$

540 quintals need

$$540 \div 45 = 12$$

✅ Answer: **12 hectares**

9.

17 m cloth = ₹493

1 m cloth

$$493 \div 17 = 29$$

27 m cloth

$$27 \times 29 = 783$$

✅ Answer: **₹783**

G. Long Answer Questions

1. A floor of dimensions 5 m × 3 m is tiled. If $\frac{1}{16}$ of the floor is tiled with coloured tiles and the remaining with white tiles, find the ratio of coloured tiles area to white tiles area.

Step 1: Find total floor area

$$\text{Area} = l \times b$$
$$= 5 \times 3$$
$$= 15m^2$$

Step 2: Find coloured tiles area

$$\text{Coloured Area} = \frac{1}{16} \times 15$$
$$= \frac{15}{16} m^2$$

Step 3: Find white tiles area

$$15 - \frac{15}{16}$$
$$= \frac{240 - 15}{16}$$
$$= \frac{225}{16} m^2$$

Step 4: Find ratio

$$\frac{15}{16} : \frac{225}{16}$$
$$15 : 225$$

Divide by 15:

$$1 : 15$$

Answer

$$\boxed{1:15}$$

---

2. Express the following ratios in simplest form

(a) 30 min : 1.5 h

Convert to same unit.

$$1.5h = 90min$$

Ratio:

$$30:90$$

Divide by 30:

$$1:3$$

Answer

$$\boxed{1:3}$$

(b) 40 cm : 1.5 m

$$1.5m = 150cm$$
$$40:150$$

Divide by 10:

$$4:15$$

Answer

$$\boxed{4:15}$$

(c) 55 paise : ₹1

$$₹1 = 100 \text{ paise}$$
$$55 : 100$$

Divide by 5:

$$11 : 20$$

Answer

$$\boxed{11 : 20}$$

(d) 500 mL : 2 L

$$2L = 2000mL$$
$$500 : 2000$$

Divide by 500:

$$1 : 4$$

Answer

$$\boxed{1 : 4}$$

---

3. Shreya and Bhoomika share ₹36 in the ratio 15 : 12. Find each one's share.

Step 1: Total parts

$$15 + 12 = 27$$

Step 2: Value of one part

$$36 \div 27$$
$$= \frac{4}{3}$$

Step 3: Shreya's share

$$15 \times \frac{4}{3}$$
$$= 20$$

Step 4: Bhoomika's share

$$12 \times \frac{4}{3}$$
$$= 16$$

Answer

$$\boxed{\text{Shreya} = ₹20}$$
$$\boxed{\text{Bhoomika} = ₹16}$$

---

4. Complete the table

Given:

$$\text{Breadth : Length} = 2:5$$

Column 1

Breadth = 10

$$\frac{2}{5} = \frac{10}{x}$$
$$2x = 50$$
$$x = 25$$

Column 2

Length = 50

$$\frac{2}{5} = \frac{x}{50}$$
$$5x = 100$$
$$x = 20$$

Column 3

Breadth = 40

$$\frac{2}{5} = \frac{40}{x}$$
$$2x = 200$$
$$x = 100$$

Completed Table

Breadth (m)	**10**	**20**	**40**
Length (m)	25	50	100

---

5. Check whether the following are in proportion

A proportion means:

$$a:b = c:d$$

or

$$\frac{a}{b} = \frac{c}{d}$$

(a)

25 cm : 1 m and ₹40 : ₹160

Convert:

$$1m = 100cm$$
$$25:100 = 1:4$$
$$40:160 = 1:4$$

Ratios equal.

Answer

$$\boxed{\text{In Proportion}}$$

Middle terms = 100, 40

Extreme terms = 25, 160

(b)

39 : 65 and 6 : 10

$$39:65 = 3:5$$
$$6:10 = 3:5$$

Equal.

Answer

$$\boxed{\text{In Proportion}}$$

Middle terms = 65, 6

Extreme terms = 39, 10

(c)

2 kg : 80 kg and 25 g : 625 g

$$2:80 = 1:40$$

$$25:625 = 1:25$$

Not equal.

Answer

$$\boxed{\text{Not in Proportion}}$$

(d)

200 mL : 2.5 L and ₹4 : ₹50

Convert:

$$2.5L = 2500mL$$
$$200:2500$$
$$= 2:25$$
$$4:50$$
$$= 2:25$$

Equal.

Answer

$$\boxed{\text{In Proportion}}$$

Middle terms = 2500, 4

Extreme terms = 200, 50

Competency-Based Questions

A. Assertion–Reason Questions

1.

Assertion (A): The simplest form of 85 : 68 is 5 : 4.

85 = 5 × 17

$68 = 4 \times 17$

$$85:68 = \frac{85}{17}:\frac{68}{17} = 5:4$$

Assertion is **True**.

Reason (R): To express a ratio in simplest form, divide both terms by their HCF.

This statement is also **True**.

Reason correctly explains Assertion.

✅ **Answer: (a) Both A and R are true and R is the correct explanation of A.**

2.

Investment ratio:

$$75000:25000 = 3:1$$

Profit = ₹4000

Monika's share:

$$4000 \times \frac{3}{4} = 3000$$

Assertion says ₹1000.

❌ Assertion is **False**.

Reason is a correct formula for division in a ratio.

✅ Reason is **True**.

✅ **Answer: (d) A is false but R is true.**

3.

If 7, 14, 15 and x are in proportion:

$$7:14 = 15:x$$
$$7x = 14 \times 15$$
$$x = 30$$

Assertion is **True**.

Reason:

If a, b, c, d are in proportion,

$$ad = bc$$

This is true and is exactly what we used.

✅ **Answer: (a)**

4.

Check:

$$8:32 = 1:4$$
$$6:36 = 1:6$$

Ratios are not equal.

Assertion is **False**.

Reason:

"If two ratios are equal then it is called proportion."

This is True.

✅ **Answer: (d)**

B. Case Study Based Questions

1.

Total members = 100

Carom = 20

Table Tennis = 24

Badminton = 16

No game:

$$100 - (20 + 24 + 16)$$
$$= 100 - 60$$
$$= 40$$

(i) Carom : Table Tennis

$$20:24$$
$$= 5:6$$

✅ **Answer: 5 : 6**

(ii) Badminton : Carom

$$16:20$$
$$= 4:5$$

✅ **Answer: 4 : 5**

(iii) Table Tennis : Badminton

$$24:16$$
$$= 3:2$$

✅ **Answer: 3 : 2**

(iv) Badminton : No game

$$16:40$$
$$= 2:5$$

✅ **Answer: 2 : 5**

2.

Total newspapers = 312

English = 216

Hindi:

$$312 - 216 = 96$$

(i) English : Hindi

$$216 : 96$$

Divide by 24

$$= 9 : 4$$

✅ **Answer: 9 : 4**

(ii) Hindi : Total

$$96 : 312$$

Divide by 24

$$= 4 : 13$$

✅ **Answer: 4 : 13**

(iii) English : Total

$$216 : 312$$

Divide by 24

$$= 9 : 13$$

✅ **Answer: 9 : 13**

Maths Booster – Crossword Puzzle

Across

1. An equality of two ratios is called a
PROPORTION

4. In a ratio, the second term is called the
CONSEQUENT

6. In a ratio, the first term is called
ANTECEDENT

7. In any proportion, the ratio of first and second quantities is ________ to the ratio of the third and fourth quantities.
EQUAL

Down

2. A comparison of two quantities of the same kind is called a
RATIO

5. In a proportion, the first and fourth terms are called
EXTREMES

3. In a proportion, the second and third terms are called
MEANS